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Automatic Monitoring and Intervention in Linear Gaussian State-Space Models: A Bayesian Approach

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Abstract. An automatic monitoring and intervention algorithm that permits the supervision of very general aspects in an univariate linear gaussian state space model is proposed. The algorithm makes use of a model comparison and selection approach within a Bayesian framework. In addition, this algorithm incorporates the possibility of eliminating earlier interventions when subsequent evidence against them comes to light. Finally, the procedure is illustrated with three empirical examples taken from the literature.

Keywords: Bayes Factor, Monitoring and Intervention, Model Comparison, Model Selection, Linear State-Space Models, Kalman Filter.

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1. Introduction

Since the seminal work of Harrison and Stevens (1976) and the publication of West and Harrison's book (1st Ed. 1989, 2nd Ed. 1997) (hereafter WH), the Bayesian analysis of the linear state space models (called Dynamic Linear Models by WH) has become a very useful tool for the analysis of univariate and multivariate time series. One of the reasons for their success is the parameter dynamism of these types of models, which allows them to adapt to the changing circumstances in which the series evolves. In addition, the sequential Bayesian methodology used in parameter estimation and forecasting allows the analyst to naturally incorporate subjective information from outside the domain of the data. Another important feature of the methodology is the sequential monitoring and intervention process described by WH in their Chapter 11. These authors state that there are two types of interventions: feed-forward and feed-back. Feed-forward interventions are subjective and anticipatory in nature and allow the analyst to incorporate external information into the model. Feed-back interventions are corrective and they are made when deterioration in the model performance has been detected. In order to carry out this last type of intervention, WH use a sequential monitoring algorithm of the one-step ahead forecast errors by using CUSUM type statistics that allow them to analyse the existence of level and variance changes. Furthermore, these authors propose a semi-automatic intervention procedure based on the diminishing of discount factors (see Section 2) associated to the building of the evolution matrices of the different model components.

However, the whole process suffers from the following drawbacks:

a) The type of aspects to be monitored (level and/or variance changes in the standardised one-step ahead forecast errors) and the kind of interventions to be carried out are very limited. Thus, for instance, if a change in the mean level of the forecast errors is detected, it is difficult to discern, using this information, whether the deterioration is due to a change in level, slope, regression coefficient or seasonal pattern or if it is caused by several of these changes occurring in the same period.

b) It does not eliminate earlier interventions when subsequent evidence against them comes to light. For this reason the final obtained models tend to be very non parsimonious.

c) The default form of intervention, even though it can be effective from a predictive viewpoint, it may not be the most adequate because it does not take into account the model components which have been responsible for the detected deterioration. In addition, from an explanatory point of view, this kind of intervention does not provide an explicit estimation of the intervention effect.

In this paper we consider all these aspects and propose an algorithm based on the comparison of the standard model with a family of alternative models similar to that considered in De Jong and Penzer (1998), which incorporate in their expression the possible interventions to be made. Furthermore, the proposed procedure reflects the uncertainty associated with the model comparison and selection process in the spirit of Occam's window proposed in Madigan and Raftery's paper (1994) and the Multi-Process Models of West and Harrison (1997).

The above family is very general and it easily incorporates a great number of different types of possible interventions to be monitored into the model. In addition, we also provide a reasonable way of building the prior distributions of the intervention parameters by using the information available at each instant. Needless to say, the algorithm is flexible enough to allow the analyst to incorporate his opinions so that he can actively intervene in the process of decision making.

The rest of this paper is organised as follows. The Linear Gaussian State Space model considered in the paper and the monitoring and intervention process proposed by WH are both described in **Section 2**. The proposed automatic monitoring and intervention algorithm is considered in **Section 3. Section 4** is devoted to an application of the proposed methodology through an analysis of four empirical examples taken from the literature. Finally, **Section 5** closes the paper with a review of the main conclusions. The mathematical proofs of the results derived from the paper are relegated to an Appendix.

2. Establishing the problem

Before presenting the model we first introduce a notation that we will be used throughout the paper. From here on, $\mathbf{0}_a$ denotes the ax1 null vector, $\mathbf{1}_a$ denotes the ax1 vector with all its components being 1, $\mathbf{i}_{a,b}$ denotes the ax1 vector with its b^{th}-component being 1 and the rest being 0, $\mathbf{I}_a$ denotes the axa identity matrix and, finally, $\mathbf{0}_{axb}$ denotes the axb null matrix. Finally $N_p(m,A)$ is the p-multivariate normal distribution with vector mean m and covariance matriz A and G(p,a) the gamma distribution with mean p/a.

The Model

Let $Y = \{Y_t, t = 1,..,T\}$ be a univariate time series, and $\forall t$ let $D_t = \{Y_t, D_{t-1}\}$ be the available information in period t.

The model considered in this paper is the linear gaussian state space model given by the following expressions:

Observation equation: $Y_t = F_t^{'}\theta_t + v_t \qquad v_t \sim N(0, V)$ (1)

Evolution equation: $\theta_t = G_t\theta_{t-1} + w_t \qquad w_t \sim N_{p_t}\left(0_{p_t}, VW_t^*\right)$ (2)

Initial information: $(\theta_0 \mid D_0, V) \sim N_{p_0}(m_0, VC_0^*)$ (3)

$$(\phi \mid D_0) \sim G\left\{\frac{n_0}{2}, \frac{n_0 S_0}{2}\right\} \text{ where } \phi = \frac{1}{V} \quad (4)$$

where $\{\theta_t\ (p_t x1);\ t=0,..,T\}$ denotes the state vectors containing the systematic components of the series evolution (trends, seasonalities, cycles, regression coefficients, etc); $\{F_t\ (p_t x1);\ t=1,...,T\}$ and $\{G_t\ (p_t x p_{t-1});\ t=1,...,T\}$ are known matrices, called the regression and evolution matrices of the model, respectively, and they are determined by those components (see examples further down and WH, Chapter 5 for more details); $\{v_t, t=1,\ldots, T\}$ (called observational errors) and $\{w_t, t = 1,.., T\}$ (called evolution errors) are internally and mutually independent, and independent of the prior distributions $(\theta_0 \mid D_0, \phi)$ and (ϕ / D_0); and m_0, C_0^*, n_0, S_0 are known constants and matrices determined by the analyst's information on the initial period t=0.

Estimating the model: the Kalman filter

The estimation of the state parameters of the model (1)-(4) is carried out in a Bayesian sequential way by using the recursive equations given in Theorem 1 which constitute the so-called Kalman filter, which enables us to recursively obtain the posterior distributions of θ_t.

Theorem 1.- In the model (1)-(4), if the posterior distribution $(\theta_{t-1} \mid D_{t-1},V)$ is $N(m_{t-1}, V C^*_{t-1})$, the one-step forecast and the posterior distribution conditional on V are given, for each t, as follows:

Prior for θ_t: $(\theta_t \mid D_{t-1},V) \sim N_{p_t}\left(a_t, VR^*_t\right)$ with $a_t = G_t m_{t-1}$ and $R^*_t = G_t C^*_{t-1} G'_t + W^*_t$

One-step forecast: $(Y_t \mid D_{t-1},V) \sim N(f_t, V Q^*_t)$ with $f_t = F'_t a_t$ and $Q^*_t = 1 + F'_t R^*_t F_t$

Posterior for θ_t: $(\theta_t \mid D_t,V) \sim N_{p_t}(m_t, V C^*_t)$ with $m_t = a_t + A_t u_t$ and $C^*_t = R^*_t - A_t A'_t Q^*_t$

where $A_t = R^*_t F_t / Q^*_t$ and $u_t = y_t - f_t$

Distribution for $\phi = V^{-1}$: $\left(\phi / D_{t-1}\right) \sim G\left\{\frac{n_{t-1}}{2}, \frac{n_{t-1}S_{t-1}}{2}\right\}$

$$\left(\phi / D_t\right) \sim G\left\{\frac{n_t}{2}, \frac{n_t S_t}{2}\right\}$$

where $n_t = n_{t-1} + 1$ and $S_t = \frac{1}{n_t}\left(n_{t-1}S_{t-1} + \frac{u_t^2}{Q^*_t}\right)$. ■

Remark 1.-

a) There are recursive algorithms to carry out the construction of the predictive distributions $Y_{t+k}|D_t$ $k \geq 1$ as well as the calculation of the retrospective posterior distributions $\theta_t|D_T$ $t \leq T$. The details of these algorithms can be seen in WH, Chapter 4, Section 4.7.

b) If the hypothesis of normality is removed the above mentioned expressions give rise to the best Linear Bayes' estimators of $\{\theta_t; t=1,...,T\}$ under quadratic loss (see WH, Section 4.9).

c) The covariance matrices of the evolution equation $\left\{W^*_t; t = 1,..., T\right\}$ are usually unknown and very difficult to determine. WH propose estimating them from $\{G_t C_{t-1} G'_t; t=1,...,T\}$ (see Theorem 1 for the expression of C_t) by using discount factors, $0 < \gamma_{m,t} \leq 1$, that quantify the loss of information produced in relation to the different components of the parameter vectors $\{\theta_t; t=1,...,T\}$ when passing from one time period to the next, in such a way that, the more close to 0 (resp. to

1) are its values, the bigger (resp. smaller) is the loss of information and the smaller (resp. bigger) is the influence of the more remote observations in the estimation of θ_t. ■

Monitoring and Intervention

An important aspect of the WH's methodology consists of the monitoring and intervention process which assess, in a sequential way, the adequacy of the model (1)-(4) to the series observed data, in order to incorporate feed-forward or feed-back interventions if needs be. The feed-forward interventions are anticipatory and subjective, in nature, and they allow the analyst to incorporate information in the model from outside the domain of the data. WH show, in their Chapter 11 how to carry out these kinds of interventions keeping up the structure of the model equations (1)-(4). However, interventions are usually retrospective, that is to say, they are carried out when inadequacies of the model to the more recent observations are detected. In order to carry out this monitoring process, WH use the most recent one-step forecast errors $\{u_i ; i=r,...,t\}$ $1\leq r\leq t\leq T$ which are calculated by using the expressions given in Theorem 1. The mathematical tool taken by WH is the Bayes' factor for model M_0 versus model M_A and it is given by:

$$BF_{M_A}^{M_0}(r,t)=\frac{P(u_r,u_{r+1},...,u_t/D_{r-1},M_0)}{P(u_r,u_{r+1},...,u_t/D_{r-1},M_A)} \qquad (5)$$

where M_0 is the standard model subject to continual assessment, M_A is an alternative model that is introduced to analyse the existence of some kind of deterioration, $P(u_r,u_{r+1},\ldots,u_t \mid D_{r-1}, M_i)$ is the predictive density of the one-step ahead forecast errors $u_r,u_{r+1},\ldots,u_t$ obtained from model M_i with $i \in \{0,A\}$, t is the current period and r is the period in which model M_0 is suspected to have experienced the shock described by the alternative model M_A. WH calculate $BF_{M_A}^{M_0}(t)=\min_{1\leq r\leq t} BF_{M_A}^{M_0}(r,t)$ and they establish a threshold $0<\tau<1$ in such a way that if $BF_{M_A}^{M_0}(t)\leq\tau$ they conclude that the analysed deterioration is significant, so the need for some form of intervention is paramount.

Taking into account that

$$BF_{M_A}^{M_0}(r,t)=\frac{P(e_r,e_{r+1},...,e_t/D_{r-1},M_0)}{P(e_r,e_{r+1},...,e_t/D_{r-1},M_A)}$$

where $\left\{ e_i = \frac{u_i}{\sqrt{S_{i-1}Q_i^*}}; i = r,..., t \right\}$ are the standardised one-step ahead forecast errors of the model M_0, the standard model used by WH is $\{e_i; r,...,t\}$ IID N(0,1) and the alternative models are M_1:$\{e_i; r,...,t\}$ IID N(μ,1); M_2:$\{e_i; r,...,t\}$ IID N(-μ,1) and M_3:$\{e_i; r,...,t\}$ IID N(0,k) with μ>0 and k > 1, in order to detect mean and/or variance changes in the model. They also propose to reduce the discount factors of the model components $\gamma_{m,t}$ with the aim of making the model more adaptive to new observations.

As we have already mentioned in the Introduction, in our view this procedure suffers from the drawbacks a), b) and c). In order to alleviate these, in the following Section we propose a new monitoring and intervention algorithm.

3. Algorithm of Simultaneous Detection of Several Shock Types

The basic idea of the proposed algorithm consists in considering an alternative family of models more general than that used by WH, which incorporates in their expression the possible interventions to be analysed and also facilitates the recursive calculation of the Bayes' factors (5).

Description of the family of alternative models

The family of alternative models is similar to that considered in De Jong and Penzer (1998) and is given by the following equations:

Observation equation: $Y_t = F_t'\theta_t + X_t'\delta + v_t \quad v_t \sim N(0, V)$ (6)

Evolution equation: $\theta_t = G_t\theta_{t-1} + H_t\delta + w_t \quad w_t \sim N_{p_t}\left(0_{p_t}, VW_t^*\right)$ (7)

Initial information: $(\theta_0 \mid D_0, V) \sim N_{p_0}(m_0, VC_0^*)$ (8)

$(\delta \mid D_0, V) \sim N_s\left(\lambda_0^*, Vq_0^*\right)$ (9)

$\left(\phi \mid D_0\right) \sim G\left\{\frac{n_0}{2}, \frac{n_0 S_0}{2}\right\} \quad \phi = V^{-1}$ (10)

with observational errors $\{v_t, t,\ldots, T\}$ and evolution errors $\{w_t, t = 1,.., T\}$ internally and mutually independent, and independent of $(\theta_0 \mid D_0, V)$, $(\delta \mid D_0, V)$ and $\left(\phi \mid D_0\right)$; m_0, C_0^*,

n_0, S_0, λ^*_0, q^*_0, $\left\{ W^*_t; t = 1,..., T \right\}$ are known constants and matrices given by the standard model (1)-(4) and δ (sx1) is a vector of shocks which affect the observation and the evolution equations. The effect of these shocks depends on matrices $\{X_t$ sx1-matrix, t = 1,…,T} and $\{H_t$ $p_t$xs-matrix, t = 1,…,T}, which are called the *shock design matrices*. The *shock magnitude* is determined by the s components of the vector δ. By specifying these matrices, it is possible to model different types of deterioration that can affect the evolution of the time series Y. Let us give some examples.

Example 1.- In a *local linear trend model* given by:

$$Y_t = F_t^{'}\theta_t + v_t \qquad \text{with } v_t \sim N(0, V)$$

$$\theta_t = G_t\theta_{t-1} + w_t \qquad \text{with } w_t \sim N_2\left(0_2, VW_t^*\right)$$

where $F_t = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, G_t = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}, \theta_t = \begin{pmatrix} \mu_t \\ \beta_t \end{pmatrix}$, μ_t is the current level and β_t is the current slope of the series Y at time t.

- A single outlying value at time j can be modelled by taking:

$$s=1,\ X_t = \begin{cases} 1 & t = j \\ 0 & t \neq j \end{cases} \text{ and } H_t = \mathbf{0}_2\ \forall t$$

- A level change at time j can be modelled by taking:

$$s=1,\ X_t = 0\quad \forall t \quad \text{and} \quad H_t = \begin{cases} \mathbf{i}_{2,1} & t = j \\ \mathbf{0}_2 & t \neq j \end{cases}$$

- A slope change at time j can be modelled by taking:

$$s=1,\ X_t = 0\quad \forall t \quad \text{and} \quad H_t = \begin{cases} \mathbf{i}_{2,2} & t = j \\ \mathbf{0}_2 & t \neq j \end{cases}$$

- A level and slope change at time j can be modelled by taking:

$$s=2,\ X_t = \mathbf{0}_2\ \forall t \text{ and } H_t = \begin{cases} \mathbf{I}_2 & t = j \\ \mathbf{0}_{2x2} & t \neq j \end{cases}. \qquad \blacksquare$$

Example 2.- In a form-free seasonal effects model (see WH Section 8.4) where $F_t^{'} = (1, \mathbf{0}_{p-1}^{'})$, $G_t = \begin{pmatrix} \mathbf{0}_{p-1} & I_{p-1} \\ 1 & \mathbf{0}_{p-1}^{'} \end{pmatrix}$, $W_t^*\, 1_p = \mathbf{0}_p$ and the state vector θ_t verifying $1_p^{'}\,\theta_t = 0\ \forall t$, a change in the seasonal pattern at time j can be modelled by taking s = p-1, $X_t = \mathbf{0}_{p-1}$

$\forall t$ and $H_t = \begin{cases} \begin{pmatrix} \mathbf{I}_{p-1} \\ -\mathbf{1}'_{p-1} \end{pmatrix} & t = j \\ \mathbf{0}_{px(p-1)} & t \neq j \end{cases}$ where $-\mathbf{1}_{p-1}$ is a (p-1) x 1 vector with all its components being (–1). ■

Example 3.- In a *dynamic regression linear model* given by:

$$Y_t = F_t'\theta_t + v_t \qquad \text{with} \quad v_t \sim N(0, V)$$

$$\theta_t = \theta_{t-1} + w_t \qquad \text{with} \quad w_t \sim N_2\left(0_2, VW_t^*\right)$$

where $F_t' = (X_{1t}, ..., X_{pt})'$ are the explanatory time series and $\theta_t' = \left(\beta_{1t}, ..., \beta_{pt}\right)'$ are the regression coefficients; a change in the regression coefficient of the k^{th} explanatory variable on period j can be modelled by taking:

$$s=1, X_t = 0 \quad \forall t \quad \text{and} \quad H_t = \begin{cases} \mathbf{i}_{p,k} & t = j \\ \mathbf{0}_p & t \neq j \end{cases}. \qquad ■$$

Remark 2.- Note that the model (6)-(10) is perfectly adapted to the modelling principle by superposition of components described in WH, Chapter 6, Section 6.2. It is possible to model shocks affecting to each one of the model components by choosing the matrices X_t and H_t in an appropriate form. In this way, for example, in a model constructed by the superposition of a local linear trend model (example 1) and a form-free seasonal effects model (example 2) it is possible to analyse shocks that affect to the model trend or to the model seasonality separately adapting, in an appropriate way, the form of the matrices X_t and H_t described in the examples 1 and 2. ■

Calculating the Bayes' factor

Once the alternative model has been described, in this Sub-section we calculate the Bayes' factor (5) taking the model given by (1)-(4) as the standard model M_0 and the model given by (6)-(10) as the alternative model M_A.

The following Theorem, the proof of which is contained in the Appendix, obtains the distribution of the one-step ahead forecast errors $\{u_i; i=1,...,t\}$ calculated by way of the standard model M_0 given by (1)-(4), when the model M_A, given by (6)-(10), is the Data Generating Process (DGP).

Theorem 2.- If the model M_A defined by (6)-(10) is true, then the distribution conditional on V of the one-step ahead forecast errors u_t will be given by:

$$(u_t \mid D_{t-1}, V, M_A, \delta) \sim N(x_t^{'}\delta, V Q_t^{*}) \quad \text{with } x_t^{'} = r_t^{'} - g_t^{'} \quad t \geq 1$$

where $r_t^{'} = F_t^{'} d_t + X_t^{'}$ with $d_t = \begin{cases} G_t d_{t-1} + H_t & \text{if } t \geq 1 \\ \mathbf{0}_{p_0 xs} & \text{if } t = 0 \end{cases}$ and $g_t^{'} = \begin{cases} F_t^{'} b_t & \text{if } t \neq 1 \\ \mathbf{0}_{1xs} & \text{if } t = 1 \end{cases}$

with $b_t = \begin{cases} G_t c_{t-1} & \text{if } t \neq 1 \\ \mathbf{0}_{p_t xs} & \text{if } t = 1 \end{cases}$ and $c_t = b_t + A_t(r_t^{'} - g_t^{'})$ where Q_t^{*} and A_t are given in Theorem 1. ■

Remark 3.- Note that the existence of a shock δ causes a bias $x_t^{'}\delta$ in the one-step ahead forecast errors of the standard model $u_t = Y_t - f_t$ in relation to the value predicted with the standard model. This bias is calculated from $r_t^{'}$, the effect of the shock in Y_t and from $g_t^{'}$, the effect of the shock on the prediction. ■

In order to calculate the Bayes' factor (5), the following Theorem gives the predictive densities for one-step ahead forecast errors under M_0 and M_A, respectively.

Theorem 3.- Let $1 \leq r \leq t \leq T$.

a) If DGP is M_0 given by (1)-(4) then, it is verified that:

$$P(u_r, u_{r+1}, \ldots, u_t \mid D_{r-1}, M_0) = \left(\prod_{i=r}^{t} 2\pi Q_i^{*} \right)^{-1/2} \frac{\Gamma\left(\frac{n_t}{2}\right)}{\Gamma\left(\frac{n_{r-1}}{2}\right)} \left(\frac{n_{r-1} S_{r-1}}{2} \right)^{\frac{n_{r-1}}{2}} \left(\frac{n_t S_t}{2} \right)^{-\frac{n_t}{2}}$$

where Q_i^{*} is given in Theorem 1, $S_j = \frac{1}{n_j}\left(n_0 S_0 + \sum_{i=1}^{j} \frac{u_i^2}{Q_i^{*}} \right)$ with $n_j = n_0 + j$ (11)

b) If DGP is M_A given by (6)-(10), then it is verified that:

$$P(u_r, u_{r+1}, \ldots, u_t \mid D_{r-1}, M_A) = \left(\prod_{i=r}^{t} 2\pi Q_i^{*} \right)^{-1/2} \left(\frac{|q_t^{*}|}{|q_{r-1}^{*}|} \right)^{1/2} \frac{\Gamma\left(\frac{n_t}{2}\right)}{\Gamma\left(\frac{n_{r-1}}{2}\right)} \left(\frac{n_{r-1} S_{r-1}^{*}}{2} \right)^{\frac{n_{r-1}}{2}} \left(\frac{n_t S_t^{*}}{2} \right)^{-\frac{n_t}{2}}$$

with

$$\lambda_j^* = E(\delta \mid V, D_j, M_A) = q_j^* \left\{ q_0^{*-1}\, \lambda_0^* + \sum_{i=1}^{j} \frac{x_i' u_i}{Q_i^*} \right\} \qquad (12)$$

$$q_j^* = V(\delta \mid V, D_j, M_A) = \left\{ q_0^{*-1} + \sum_{i=1}^{j} \frac{x_i' x_i}{Q_i^*} \right\}^{-1} \qquad (13)$$

$$S_j^* = \frac{1}{n_j}\left(n_j S_j + \lambda_0^{*'} q_0^{*-1} \lambda_0^* - \lambda_j^{*'} q_j^{*-1} \lambda_j^* \right); \qquad j=0,\ldots,T \qquad (14)$$

where n_j and S_j are given in (11). ■

Using this Theorem, the following Corollary is easily obtained.

Corollary 1.- Let $1 \le r \le t \le T$. If M_0 and M_A are given by (1)-(4) and (6)-(10), respectively, then it is verified that:

$$BF_{M_A}^{M_0}(r,t) = \left(\frac{|q_{r-1}^*|}{|q_t^*|} \right)^{1/2} \left(\frac{n_t S_t^*}{n_t S_t} \right)^{\frac{n_t}{2}} \left(\frac{n_{r-1} S_{r-1}}{n_{r-1} S_{r-1}^*} \right)^{\frac{n_{r-1}}{2}} \qquad (15)$$

where n_j and S_j, q_j^* and S_j^* are defined in (11), (13) and (14), respectively. ■

Remark 4.-

a) Note that from (11), (12), (13) and (14) it is verified that:

$$n_j S_j = n_{j-1} S_{j-1} + \frac{u_j^2}{Q_j^*}\,;\; q_j^{*-1} = q_{j-1}^{*-1} + \frac{x_j' x_j}{Q_j^*}\,;$$

$$q_j^{*-1}\lambda_j^* = q_{j-1}^{*-1}\lambda_{j-1}^* + \frac{x_j' u_j}{Q_j^*}\;;\; n_j S_j^* = n_j S_j + \lambda_0^{*'} q_0^{*-1} \lambda_0^* - \lambda_j^{*'} q_j^{*-1} \lambda_j^*$$

in such a way that, this latent recursivity simplifies the calculation of the Bayes' factor (15).

b) In the algorithm described in Sub-section 3.3 and in the examples analysed in Section 4, the alternative model set considered in order to carry out the monitoring and intervention process typically consists of models given by (6)-(10) with $\{X_v = 0_{1xs}\,;\, H_v = \mathbf{0}_{p_v xs}\,;\, v<r\}$ (see examples 1, 2 and 3 in Sub-section 3.1). In these cases, the Bayes' factor expression (15) is simplified to:

$$\left(\frac{|q_0^*|}{|q_t^*|}\right)^{1/2}\left(\frac{n_t S_t^*}{n_t S_t}\right)^{\frac{n_t}{2}}$$

thereby reducing the computational effort of the algorithm. ■

Our aim is to build a procedure that jointly analyses the existence of both outliers and structural changes. In order to do so, the following Sub-section provides an algorithm to carry out this task.

The Algorithm

The algorithm is based on the comparison of the standard model with a set of alternative models given by expressions (6)-(10) that are constructed in order to analyse the existence of possible deteriorations in the standard model described by the design matrices X_t and H_t. The comparison is carried out in a sequential way by means of the Bayes' factor (15) determining, in each period, which interventions must be incorporated into the model and which must be removed from it. The process ends with a search for a parsimonious model representation, once all the series observations have been analysed.

Following the philosophy of changing only those aspects which need to be changed, we will choose the design matrices X_t and H_t in accordance with those aspects of the model that the analyst wants to monitor (trend level, trend slope, seasonal pattern, regression coefficient, etc.). Furthermore, we will specify the period $t_\ell \in \{1,...,T\}$ from which it is estimated that the intervention should be incorporated into the model. In this way, each possible intervention will be given by a couple $\ell = (i_\ell, t_\ell)$ where i_ℓ is the type of intervention (isolated outlier, level change, slope change, etc). When this intervention is incorporated into the standard model, it gives rise to an alternative model M_ℓ given by (6)-(10) where $\{X_t = \mathbf{0}_s$ and $H_t = \mathbf{0}_{p_t x s}$ $t \leq t_\ell - 1\}$ and the matrices $\{X_t, H_t;\ t=t_\ell,...,T\}$ are determined by the type of intervention i_ℓ. The examples 1 to 3 given in Sub-section 3.1 are particular cases of this kind of models.

The algorithm consists of four steps: i) Start, ii) Location and incorporation of the interventions to be carried out, iii) Elimination of earlier interventions and iv) Search for a parsimonious model representation. We will now offer a detailed description of each step, using the following terminology:

Let OI be the **types of ordinary interventions** whose existence the analyst wishes to monitor continually. These types of interventions allude to possible model progressive deteriorations due to the existence of shocks which affect one, or several, model components; and they are not usually revealed by abnormally high standardised errors.

Let EI be the **types of extraordinary interventions** which the analyst considers can be feasibly employed when the more recent standardised one-step forecast error e_t is great. Typically EI = OI $\cup$ {isolated outlier}.

Let DI be the set of **doubtful interventions**, i.e. interventions which suggest that their inclusion in the model seems to be advisable but which are waiting for the accumulation of evidence that will allow the monitor to discern whether they have to be incorporated into the model or, alternatively, have to be completely removed.

Finally, let FI be the set of **fixed interventions**, i.e. interventions incorporated into the standard model at a particular moment in time, but with the possibility of being removed if evidence is accumulated against them.

Furthermore, we will need to specify the following elements:

r_{min}: minimum number of observations necessary in order to make the decision of whether to turn a doubtful intervention into a fixed intervention.

e_{min}**:** minimum error used to detect single outliers.

t_{min}**:** initial number of observations that the analyst remove, in order to reduce the effect of the prior distribution.

and the following thresholds:

τ_1**:** threshold to decide if an intervention is to be removed.

τ_2**:** threshold to decide if a doubtful intervention is to be incorporated into the model as a fixed intervention.

τ_3**:** threshold to carry out the parsimony step in which fixed interventions that are not significant enough are removed.

With these elements, the automatic monitoring and intervention algorithm is comprised of the following steps:

Step 1: START

The initial standard model given by (1)-(4) is established to describe the observed series evolution and is denoted by M_0. In addition, we specify OI, EI as well as the parameters $t_{min} \in \{1,...,T\}$, $r_{min} \in \mathbf{N}$, $e_{min} \geq 0$, $\tau_1 \in [0,1]$, $\tau_2 \in [0,1]$, $\tau_3 \in [0,1]$.

In order to reduce the effect of the prior distribution, apply the Kalman filter during the first t_{min} observations. After this learning period, set DI = $\{(i,t_{min}): i \in OI\}$, FI = $\varnothing$ and $t = t_{min}$. Go to Step 2.

Step 2: LOCATION AND PROCESSING OF DOUBTFUL INTERVENTIONS

In this step, we calculate the possible interventions to be carried out in each period of time t. These interventions can be either due to the appearance of an unusually large standardized one-step ahead forecast error e_t (extraordinary interventions) or to the onset of a new ordinary intervention that replaces one that has been eliminated earlier. In addition, the doubtful interventions which are incompatible with the available data are removed and the necessity of incorporating some doubtful interventions into the model is analysed. An intervention ℓ is incorporated into the model if it significantly improved the standard model and, furthermore, if it is supported by a number of observations larger than or equal to r_{min}. The steps to be carried out are as follows:

a) Location of extraordinary interventions.

   If $|e_t| \geq e_{min}$ set $DI = DI \cup \{(i,t): i \in EI\}$.

b) Ordinary interventions that have to be removed.

   Set $OI^* = \{i_\ell \in OI : \ell=(i_\ell,t_\ell) \in DI \text{ and } CBF_{M_\ell}^{M_0}(t_\ell, t) > \tau_1\}$.

c) Elimination of doubtful interventions.

   Set $DI = DI - \{\ell \in DI: CBF_{M_\ell}^{M_0}(t_\ell, t) > \tau_1\}$.

d) Updating of ordinary interventions.

   Set $DI = DI \cup \{(i,t+1): i \in OI^*\}$.

e) Analysis of the need to incorporate a new fixed intervention.

   Calculate $\ell_{min} \in DI$, such that

$$CBF_{M_{\ell_{min}}}^{M_0}(t_{\ell_{min}}, t) = \min_{\ell \in DI^*} CBF_{M_\ell}^{M_0}(t_\ell, t)$$

where $DI^* = DI - \{\ell \in DI: t_\ell - t > r_{min}\}$.

If $CBF_{M_{\ell_{min}}}^{M_0}\left(t_{\ell_{min}}, t\right) < \tau_2$, then

$FI = FI \cup \{\ell_{min}\}$, $DI = DI - \{\ell_{min}\}$

Set $M_0 = M_{\ell_{min}}$ and repeat Step 2;

otherwise go to Step 3.

Step 3: PROCESSING OF FIXED INTERVENTIONS.

In this step, interventions previously incorporated into the standard model are examined in order to analyse if it is possible to eliminate some of them. It consists of the following stages:

a) Analysis of the need to eliminate a fixed intervention.

Calculate $\ell_{max} \in FI$, such that

$$CBF_{M_0}^{M_{-\ell_{max}}}\left(t_{\ell_{max}}, t\right) = \max_{\ell \in FI} CBF_{M_0}^{M_{-\ell}}\left(t_\ell, t\right)$$

where $M_{-\ell}$ denotes the model which eliminates the intervention ℓ from the standard model.

If $CBF_{M_0}^{M_{-\ell_{max}}}\left(t_{\ell_{max}}, t\right) \geq \tau_1$ then $FI = FI - \{\ell_{max}\}$ and set $M_0 = M_{-\ell_{max}}$. Go to Step 2.

b) Processing of a new observation.

Set $t = t + 1$ and calculate the standardised one-step ahead forecast error et by means of the Kalman filter. If $t < T$ then, go to Step 2; otherwise go to Step 4.

Step 4: SEARCH FOR A PARSIMONIOUS MODEL REPRESENTATION.

Once all the observations of the series have been processed and, in order to build a parsimonious model, repeat Steps 2 and 3 with $t = T$ until FI does not change. In Step 3 the threshold τ_3 is used instead of τ_1 where the threshold τ_3 is chosen in order to obtain parsimonious models. The resultant standard model is the selected model.

Remark 5.- (Choice of the elements r_{min}, e_{min}, t_{min} and the thresholds τ_1, τ_2, τ_3)

a) It is important to choose $r_{min} > 1$ given that with only one observation it is impossible to distinguish the type of the intervention to be carried out (it can be either an outlier, the onset of a level change, the onset of a seasonal pattern change, etc). On the other hand, it is not desirable to choose too large a value, due to the need to incorporate the intervention as soon as possible. Our advice is to take r_{min} equal to the maximum of the dimensions of the parameter vectors δ associated to the types of interventions that the analyst wishes to monitor, with the aim of having at least one observation for each component of δ.

b) e_{min} is the parameter which determines the introduction of the doubtful extraordinary interventions. In our examples we have taken $e_{min} = 1.645$ with the aim of having, roughly, a 10% of probability to incorporate doubtful interventions without need.

c) The value of t_{min} depends on the number of the dynamic parameters of the standard model. If the prior distribution is improper t_{min} would take a value larger than or equal to this number with the purpose of starting the algorithm with a proper posterior distribution of these parameters. In our examples, we have taken t_{min} equal to $p_0 +1$ in order to have al least one observation per parameter.

d) Given that incorporating an intervention makes the standard model less parsimonious, a sensible choice of the threshold τ_1 is 1 (this value is used by WH or by Madigan and Raftery (1994)). The threshold τ_2 determines when an intervention has to be incorporated, that is to say, when it is significant enough. The Bayesian literature (Jeffreys (1961), Madigan and Raftery (1994), Kass and Raftery (1995)) advises to take values between 0.01 and 0.10. Some simulations described in Gargallo and Salvador (2002) suggest to take $\tau_2 = 0.05$ if a level significance of 5% is desirable and $\tau_2 = 0.01$ if a level significance of 1% is desirable. In the practical cases we describe later in the paper, we have taken $\tau_2 = 0.05$ which is also the value advised by Madigan and Raftery (1994).

e) The threshold τ_3 is used in Step 4 of the algorithm. This Step is only implemented to obtain a parsimonious model. In our empirical examples, we have taken τ_3 = 1/3, that is to say, the advised value in Kass and Raftery (1995) p. 777. ■

Remark 6.- (Choice of prior parameters λ_0^* and q_0^*)

Given that the standard model is initially true, a sensible choice of λ_0^* would be $\mathbf{0}_s$. The value of q_0^* would be determined by the model components which are affected by the intervention and the period t in which this intervention is carried out. A logical choice would be to take q_0^* equal to the block of the prior variance matrix, R_t^*, given in Theorem 1, corresponding to the components of the state vector that are affected by the shock, except if the shock is an outlier; in which case we would take $q_0^* = Q_t^*$. ■

Remark 7.- In the spirit of Madigan and Raftery (1994), the class of the alternative models corresponding to the doubtful interventions together with the standard model constitute the Occam's window which give rise to one mixture of models with posterior probabilities proportional to the corresponding Bayes' factors and which could be used to make predictions and to take decisions. This possibility has not been explicitly explored in this paper but it will be studied in future papers. ■

Selective Automatic Intervention

If the algorithm detects that a shock has taken place, the intervention method is determined by the equations of model M_A corresponding to this shock. The new standard model will be given by the equations:

$$Y_t = F_t^{*'} \theta_t^* + v_t^*$$

$$\theta_t^* = G_t^* \theta_{t-1}^* + w_t^*$$

where $F_t^{*'} = \left(F_t', X_t'\right)$, $v_t^* = v_t$, $G_t^* = \begin{pmatrix} G_t & H_t \\ 0_{sxp_{t-1}} & I_s \end{pmatrix}$, $w_t^* = \begin{pmatrix} w_t \\ 0_s \end{pmatrix}$ and $\theta_t^* = \begin{pmatrix} \theta_t \\ \delta \end{pmatrix}$.

Thus, the structure of the equations (1)-(4) is maintained, and it is possible to apply Theorem 1 in order to carry out the *on-line* estimation of the parameters, as well as, to use the algorithms described in WH, Chapter 4 for the calculation of the predictive $Y_{t+k}|D_t$ $k \geq 1$ and the retrospective $\theta_t|D_T$ $t \leq T$ distributions.

4. Empirical Illustrations

In this Section we illustrate the use of the proposed algorithm by analysing four empirical time series taken from the literature. In the considered data sets we have applied the following procedures:

i) The Kalman filter without carrying out any monitoring and intervention process, i.e., by using the equations provided in Theorem 1.

ii) The proposed algorithm with $e_{min} = 1.645$, $\tau_1 = 1$, $\tau_2 = 0.05$ and $\tau_3 = 1/3$ (denoted by GS monitor from here on).

iii) The algorithm proposed by WH in Chapter 11 (described by ourselves as WH monitor) in order to monitor mean and variance changes of the standardized one-step ahead forecast errors. The values adopted in this study have been h =1.5, k = 4 and $\tau = 0.05$, which correspond to a significance level of approximately 5%, and are the values that gave the best results in the simulations carry out by Gargallo (2001) and Gargallo and Salvador (2003). WH propose to intervene in the model by decreasing the discount factors of its parameters, with the aim of making it more adaptive to the most recent observations. In our case, whenever the monitor detects a change, we take $\delta_{m,t} = 0.1$ as the discount factor for the parameters of the trend and the seasonality, $\delta_{m,t} = 0.8$ for the regression coefficient and $\delta_{v,t} = 0.9$ for the observational variance.

These procedures are numerically compared by using:

$$\text{MAD} = \frac{1}{T - t_{min}} \sum_{t=t_{min}+1}^{T} |u_t|$$

$$\text{RMSE} = \sqrt{\frac{1}{T - t_{min}} \sum_{t=t_{min}+1}^{T} u_t^2}$$

as measures of the model predictive performance and

$$\text{LLF} = \frac{1}{T - t_{min}} \sum_{t=t_{min}+1}^{T} \log p(Y_t | D_{t-1})$$

as a measure of goodness of fit, where $u_t = Y_t - E[Y_t|D_{t-1}]$ is the one-step ahead forecast outsampling error.

This numerical analysis has been completed by the elaboration of graphics that compare the one-step forecast, $f_t = E[Y_t|D_{t-1}]$, obtained by each procedure, with the observed values of the series, accompanied, in each case, by their Bayesian 99% predictive confidence intervals $f_t \pm t_{n_{t-1}}(0.005)\sqrt{\frac{n_{t-1}}{(n_{t-1}-2)}S_{t-1}Q_t^*}$ where $t_{n_{t-1}}(\alpha)$ denotes the 1-α/2 quantile of a central Student t distribution with n_{t-1} freedom degrees and f_t, n_t, S_t and Q_t^* are given in Theorem 1[1]. In these graphs we have marked the incorporated interventions by vertical lines that indicate the moment in which these have taken place, and their type. Thus, the form of the line indicates the intervention type (unbroken line: outlier; dotted line: level change; dashed line: level and slope change; dashed and dotted line: other changes (slope, seasonal or regression coeficient change), whilst the colour indicates if there have been one (black line) or several (grey line) interventions located in the same period. Finally, and given that the monitoring algorithm of WH does not distinguish the type of shock detected, the corresponding graphics only reflect the instant at which the change is detected and the intervention applied.

Level of Lake Huron

This series has been analyized by Brockwell and Davis (2002) and provides the annual level in feet of Lake Huron (reduced by 570) in the years 1875-1972. These authors (considering various models) fit a model with constant linear trend (see Figure 2) given by the equations:

$$Y_t = \underset{(0.463)}{10.091} \; \underset{(0.008)}{-0.0216}t + u_t$$

where u_t follows a AR(2) process. With the aim of analyzing the possible existence of changes in the trend of the series, we fit a model with linear local trend ($F_t = \begin{pmatrix}1\\0\end{pmatrix}$; $G_t = \begin{pmatrix}1 & 1\\0 & 1\end{pmatrix}$ $\forall t$) with discount factors equal to 1 and prior distribution given by (3) and (4)

[1] We also analysed the behaviour of the standardised one-step ahead forecast errors $e_t = \frac{u_t}{\sqrt{Var(u_t \mid D_{t-1})}}$. The results are not shown for the sake of brevity.

with $m_0 = (11,0)'$, $C_0 = I_2$, $n_0 = 1$ and $S_0 = 1$. This prior distribution is quite diffuse, in such a way that it hardly affects the results described hereafter. We have applied the algorithm described in the previous Section in order to analyze the possible existence of single outliers, changes in level and joint changes in level and slope. Furthermore, in order to implement the monitor we have taken the parameter values indicated in ii) and $r_{min} = t_{min} = 3$. The results obtained are set out in Tables 1 and 2 and in Figures 1, 2 and 3.

We can observe (see Table 1) that the GS monitor achieves the best results in all three criteria considered in the paper. This monitor detects the existence of a change in the level and slope of the trend in 1942 and 1943, as well as two outliers in 1929 and 1964, with the second of these two corresponding to the minimum historical level of the water in that lake (see Figure 1). The obtained model estimates that until 1942 the trend comes given by :

$$Y_t = \underset{(0.222)}{10.824} - \underset{(0.008)}{0.0476}t + u_t$$

and, from 1943 onwards, by:

$$Y_t = \underset{(0.150)}{8.071} - \underset{(0.007)}{0.0094}t + u_t$$

Therefore, we can appreciate a stabilization in the water level of the lake from 1943 onwards (see Figure 2). Table 2 shows the estimated magnitude of these changes, and here we can note that all of them are significant at a 95% level.

(Insert Figures 1 and 2 and Tables 1 and 2 about here)

For its part, the WH monitor obtains better results than under the non-intervention approach (see Table 1). However, it intervenes in an excessive number of years (1891, 1908, 1925, 1929, 1944, 1958, 1970, see Figure 1) and, as a consequence, it fits a less parsimonious model, and one with a worse fit under the 3 criteria, than the model fitted by the GS monitor. This is essentially due to the fact that, in its implementation, the WH monitor does not incorporate a similar step to the 3 and 4 steps of the GS algorithm, which analyse for whether the interventions incorporated at earlier moments in time to the current one continue to be significant. As can be appreciated from Figures 2 and 3, almost all the interventions made by this monitor correspond to changes in the level and/or slope of the trend that are of only limited significance, given that the confidence bands overlap both prior to and following each change. The only significant intervention is that made in 1944 and which corresponds, in general terms, to that made by the GS monitor.

(Insert Figure 3 about here)

Series CP6

This series has been analyzed by WH (Chapter 11) in order to illustrate how to carry out feed-forward interventions and provides the monthly total sales, in monetary terms, of tobacco and related products marketed by a major company in the UK. The data runs from January 1955 to December 1959, inclusive. Figure 4 shows a series trend as a sequence of roughly linear segments, with three major changes, namely in December 1955, January 1957 and January 1958 (see WH to obtain more information on these). In our case, we suppose that there is no prospective information and we will monitor the evolution of the series by applying the three procedures described earlier. The standard model we use is the same as that used in WH, that is to say, a model with linear local trend ($F_t = (1,0)'$; $G_t = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ $\forall t$) and with the same prior distributions and the same discount factors. We have applied the algorithm described in the previous Section in order to analyze the possible existence of single outliers, changes in level and joint changes in level and slope. Furthermore, in order to implement the monitor we have taken the parameter values indicated in ii) and $r_{min} = t_{min} = 3$. The results obtained are set out in Tables 3 and 4 and in Figures 4 and 5.

We can observe (see Table 3) that the GS monitor achieves the best results in all three criteria considered in the paper. Thus, this monitor detects the most relevant changes (outlier in December 1955 and level changes in January 1957 and January 1958); in addition, it detects a change in the level and in the slope of the series in August 1958 which allows it to follow the posterior evolution in a more accurate way (see Figure 4). Table 4 shows the estimated magnitude of these changes, and here we can note that all of them are significant at a 95% level.

(Insert Figures 4 and 5 and Tables 3 and 4 about here)

On the other hand, in spite of the fact that the WH monitor obtains better results than the non-intervention approach, and that it detects the changes that took place in

December 1955 and January 1957, it does not detect the changes produced in the series around 1958. This is due to the fact that the way of intervention of this monitor does not take into account the type of the change detected and intervenes in all of the model parameters. For example, the WH monitor detects a level change in January 1957 and, as a consequence, intervenes not only in the mean level but also in the slope and in the observational variance. This provokes an excessive increase of the uncertainty associated to the on-line estimation of the model parameters and, as a consequence, a greater width of the predictive bands of the model (see Figures 4 and 5). This increase diminishes the sensitivity of the monitor to possible changes, given that the standardised errors $e_t = \frac{u_t}{\sqrt{\mathrm{Var}(u_t \mid D_{t-1})}}$ are artificially small. For this reason, the monitor does not detect, or detects too late, the changes produced throughout 1958, which results in a worse fit of the estimated model. By contrast, the GS monitor also detects this change, intervening in the mean level of the series but not in the rest of the parameters. This makes the monitor more sensitive to new changes and allows it to detect the level and slope change that took place in the middle of 1958. Therefore, the estimated trend follows the evolution of the series more accurately, providing a better fit to the data.

This effect can also be seen in the retrospective estimations of the trend obtained by the smoothing algorithm described in WH Section 4.8 (see Figure 6), whose fit to the evolution of the series is worse than that estimated by the GS monitor and with wider confidence bands.

(Insert Figure 6 about here)

Deaths and Serious Injuries in Road Accidents

The third example concerns the logarithm of the monthly number of car drivers killed or seriously injured in the United Kingdom from January 1969 to December 1984 and are taken from Harvey and Durbin (1986), who analysed this series to assess the effects of the UK seat-belt law by front seat occupants of cars which was made compulsory in February 1983. This data has been examined by many authors, such as Ng and Young (1990), Balke (1993) and Pole et al. (1994).

From Figure 7 it is relatively easy to see the general features of this time series. There is an overall level trend that seems to be broken into three separate phases with breaks at the beginning of 1974 and at the beginning of 1983, and a pronounced seasonal pattern that seems fairly consistent across the entire series. Within each of the segments 1969-1973, 1974-1982, and 1983-1984, the series exhibits apparently little movement in the trend and seasonal pattern. Therefore, we propose a model comprising a second-order polynomial model for the trend and a model with 12 form-free seasonal effects for seasonality, i.e., $\forall t\ F_t' = \left(E_{2,1}', E_{12,1}'\right)$ and G_t = diag(A,B) with $A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} \mathbf{0}_{11} & I_{11} \\ 1 & \mathbf{0}_{11}' \end{pmatrix}$ where I_{11} is the 11x11 identity matrix and $\mathbf{0}_{11}$ is the 11x1 null vector. We use an initial diffuse prior distribution and we apply the monitoring and intervention algorithm, supervising the existence of outliers and changes in level and/or slope and in the seasonal pattern. We take $t_{min} = r_{min} = 12$ and $\gamma_{m,t} = 0.99\ \forall t$ as discount factors, which presuppose a roughly constant evolution of the different model components. The obtained results are shown in Tables 5 and 6 and Figures 7 to 10.

The best results again correspond to the GS monitor (see Table 5) that detects level and slope changes in October and November 1973, January 1975 and February 1983, a level change in January 1983, as well as an outlier in August 1976 (see Figure 7). In this way, it captures the change in trend caused by the oil crisis and the posterior stabilization of the trend from 1975 onwards, as well as the effect of the compulsory seat belt law introduced in January 1983. Table 6 shows the estimation of the magnitude of these changes; in particular, we can conclude that the seat-belt law reduced the number of injuries by around 7.75% and also produced a decrease of the monthly growth rate of about 0.06%. The incorporation of these interventions substantially improves the predictive behaviour and the goodness of fit of the model, as we can appreciate from the results given in Table 5.

(Insert Figure 7 and Tables 5 and 6 about here)

The WH monitor obtains slighly better results than non-intervention, and this in spite of the fact that it particularly detects the changes that took place at the end of 1973, as well as the effect of the introduction of the seat belt law in January 1983. The reason for a worse fit to the data is again due to the form of intervention. Figures 8 and 9 show the on-line estimations of the model trend and the seasonality. We can again clearly appreciate what we commented on earlier, that is to say, the confidence bands of the estimated seasonality are wider than those corresponding to the GS monitor, given that the latter only intervenes in the trend. After each intervention, the confidence bands of these estimations increase to an excessive extent, which results in a decrease in the sensitivity of the monitor to new changes and a worse fit of the model to the data.

(Insert Figures 8 and 9 about here)

Finally, Figure 10 shows the retrospective estimations of the series trend obtained with the three procedures. We can see that the two monitors provide similar estimations in both components and that the estimated trend describes the evolution of the series more appropriately than does non-intervention. However, the width of the confidence bands of these estimations is less in the GS monitor, for the earlier-mentioned reasons.

(Insert Figure 10 about here)

Commercial Sales in the Food Industry

The third and last example corresponds to a dynamic regression model considered in WH (Section 10.3). The dependent series (Y_t) is the monetary value of total monthly sales on a standardized, deflacted scale of a widely consumed and established food product in UK markets. The independent series (X_t) is a standardized compound measure constructed by the company concerned, based on market prices, production and distributions costs and related variables. The data runs over the full six years 1976 to 1981 inclusive; thus, there are 72 observations. We have adopted a dynamic regression linear model with seasonal effects, such that $\forall t \; F_t' = (1, X_t, 1, \mathbf{0}_{11}')$, $G_t = \begin{pmatrix} I_2 & \mathbf{0}_{2x12} \\ \mathbf{0}_{12x2} & B \end{pmatrix}$ where $B = \begin{pmatrix} \mathbf{0}_{11} & I_{11} \\ 1 & \mathbf{0}_{11}' \end{pmatrix}$. The parameters of the prior distributions and the discount factors of each component of the model are the same as those in WH. In the case of the GS monitor, we monitor the presence of outliers, changes in the intercept and in the regression coefficient, as well as changes in the seasonal pattern, taking t_{min} = 14 and r_{min} = 12.

The results obtained are shown in Tables 7 and 8 and Figures 11 to 15. Table 7 displays the values obtained for each criteria and each monitor. It confirms that the best results correspond to the WH monitor in the RMSE and MAD criteria, which, paradoxically, obtains the worst results in terms of the LLF criterion.

(Insert Table 7 about here)

The explanation is again in the way of intervening. The WH monitor detects two changes in June 1977 and December 1980 (see Figure 11). However, given that it does not discriminate the type of change, it intervenes in all the model components (see, for example, Figures 12 and 13), thereby unnecessarily increasing the level of uncertainty. This decreases its sensitivity in detecting other changes, as well as the goodness of fit the model. In particular, the monitor does not detect the increase in the level that took place around May 1978, which the GS monitor manages to detect (see Figure 11), in such a way that the latter obtains a better goodness of fit of the model, given that it only intervenes in the component affected by this change (see Figures 12 and 13). Table 8 shows the magnitude of the change, and here we can note that it is clearly significant.

(Insert Figures 11, 12 and 13 and Table 8 about here)

Figures 14 and 15 show the retrospective estimations obtained by the three compared procedures. We can observe that only the GS monitor captures the effect that the independent variable exerts on the dependent variable in a significant way (see Figure 15) and, more particularly, its decrease over time. This is due to the fact that, thanks to the intervention carried out, the regression coefficient is estimated with a higher precision than that obtained with non-intervention and with the WH monitor. In this last case, we can observe that the confidence bands are again too wide to draw any conclusion about this effect.

(Insert Figures 14 and 15 about here)

Discussion

The 4 examples analyzed reveal that the monitor proposed in this paper has a capacity to detect changes similar to that of the WH monitor. The essential difference between the two lies in the way of intervening. The WH monitor, because it intervenes in all the model components, introduces excessively wide levels of uncertainty in the on-line estimation process of the parameters. This can cause a decrease in the monitor's sensitivity for detecting new changes, given that the standardized one-step ahead forecast errors e_t are too small, as well as a worse goodness of fit to the model due to the excessively wide out-sampling predictive bands.

It could be argued that this effect would be cancelled if the way of intervening were changed. However, it is not clear how this change should be carried out, given that the standardized one-step ahead forecast errors e_t are the only source of information of the WH monitor. Thus, it is difficult to determine whether the change has taken place in the trend, in the seasonality or in the regression coefficients of the model. The GS monitor does not suffer from these defects, because each intervention has an alternative model, given by (6)-(10), associated to it. This alternative indicates the way of intervening and, more particularly, what are the components in which we have to intervene, as well as how to estimate the magnitude of the changes that have taken place in the model. In addition, the algorithm allows us to retrospectively eliminate interventions that have little significance, which increases the parsimony of the fitted models and decreases the width of the out-sampling predictive intervals, thereby allowing us to take decisions in a more appropriate way.

5. Conclusions

In this paper, we have proposed a monitoring and intervention algorithm in univariate linear gaussian state space models, based on the comparison between the standard model, on the one hand, and alternative models that explicitly include the analysed interventions in their expression, on the other. Thus, we have transformed the monitoring process into a model comparison and selection problem. By doing so, the type of possible interventions is extended and it is possible to remove interventions previously made when the statistical evidence against them is sufficiently important and to estimate the size of the shocks that affect the series evolution.

The proposed methodology has been illustrated by analysing four empirical examples taken from the literature that show the generality and flexibility of the method. The algorithm offers a monitoring power similar to that of the monitor proposed by WH Chapter 11 and, furthermore, allows us to make interventions more precise than the WH monitor, which results in a better goodness of fit of the model to the data.

Although the proposed methodology has been applied to the analysis of univariate series, it is possible to extend it to the analysis of multivariate series that evolve in accordance with a linear state space models. The results of this extension will be presented in a future paper. On the other hand, the methodology has been applied to univariate time series that evolve according to the linear state space model given by (1)-(4). This model in particular, assumes that the observation error is homoscedastic. However, on some occasions (for example, in financial series) the analysed series is heteroscedastic. Future areas of research would be to extend the method so as to analyse this kind of series by providing the possibility of interventions in variance.

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Appendix

Hereafter and with the purpose of simplifying the notation we will suppose, without loss of generality, that the onset of the deterioration took place at the instant t-r+1=1. Before turning to the proof of the mathematical results presented in the paper, we will first consider the following notation:

Let Y_t^0 be the value that we would expect to observe in the series if the change had not been produced, i.e., if the DGP had been M_0; let θ_t^0 be the value that we would have of the state vector if the DGP had been M_0 and $E_i(. \mid D_{t-1}, M_j)$ the expected value calculated applying the Kalman filter corresponding to the model M_i (i = 0,A) under the assumptions that the DGP is the model M_j (j = 0,A).

In addition, we use a superindex i to represent the mean values of the distributions obtained by applying the Kalman filter corresponding to the model M_i and under the assumptions that the DGP is M_i (i = 0, A), that is to say:

$$a_t^0 = E_0(\theta_t \mid D_{t-1}, M_0) \quad \text{and} \quad a_t^A = E_A(\theta_t \mid D_{t-1}, M_A, \delta) \qquad (A.1)$$

$$f_t^0 = E_0(Y_t \mid D_{t-1}, M_0) \quad \text{and} \quad f_t^A = E_A(Y_t \mid D_{t-1}, M_A, \delta) \qquad (A.2)$$

$$m_t^0 = E_0(\theta_t \mid D_t, M_0) \quad \text{and} \quad m_t^A = E_A(\theta_t \mid D_t, M_1, \delta) \qquad (A.3)$$

Note that we do not use superindex to represent the mean value of the distributions obtained by applying the Kalman filter corresponding to the model M_0 and under the assumptions that the DGP is M_A, i.e.:

$$a_t = E_0(\theta_t \mid D_{t-1}, M_A, \delta) \tag{A.4}$$

$$f_t = E_0(Y_t \mid D_{t-1}, M_A, \delta) \tag{A.5}$$

$$m_t = E_0(\theta_t \mid D_t, M_A, \delta) \tag{A.6}$$

Before demonstrating Theorem 2, we need the following four lemmas.

Lemma 1.- If model M_A given by (6)-(10) is true, it is verified that:

$$\theta_t = \theta_t^0 + d_t\delta \quad \text{where } d_t = G_t d_{t-1} + H_t \quad \forall\, t \geq 1 \quad \text{and } d_0 = 0_{p_0 xs} \tag{A.7}$$

Proof.- The proof is made by induction. Using the previous notation, it is verified that:

$$\theta_0 = \theta_0^0 = \theta_0^0 + d_0\delta$$

that is to say, the result is true for t = 0 and, therefore, if we assume that the result is true for period t-1, then for period t we have:

$$\theta_t = G_t\theta_{t-1} + H_t\delta + w_t = G_t\theta_{t-1}^0 + G_t d_{t-1}\delta + H_t\delta + w_t = \theta_t^0 + (G_t d_{t-1} + H_t)\delta = \theta_t^0 + d_t\delta$$

with $d_t = G_t d_{t-1} + H_t$, in such a way that the proof is completed. ■

Lemma 2.- If the model M_A given by (6)-(10) is true, it is verified that:

$$Y_t = Y_t^0 + r_t'\delta \quad \text{being } r_t' = F_t' d_t + X_t' \quad \forall\, t \geq 1 \tag{A.8}$$

Proof.- From Lemma 1 it is easy to see that in period t we have:

$$Y_t = F_t'\theta_t + X_t'\delta + v_t = F_t'\theta_t^0 + F_t' d_t\delta + X_t'\delta + v_t = Y_t^0 + (F_t' d_t + X_t')\delta = Y_t^0 + r_t'\delta$$

with $r_t' = F_t' d_t + X_t'$, in such a way that the result is established. ■

Lemma 3.- If the model M_A given by (6)-(10) is true, it is verified that:

$$a_t = a_t^0 + b_t\delta \qquad f_t = f_t^0 + g_t'\delta \qquad \text{and} \qquad m_t = m_t^0 + c_t\delta \tag{A.9}$$

being:

$$b_t = \begin{cases} 0_{p_t x s} & t=1 \\ G_t c_{t-1} & t \neq 1 \end{cases} \quad \text{with } c_t = b_t + A_t(r_t^{'} - g_t^{'}) \text{ and } g_t^{'} = F_t^{'} b_t \tag{A.10}$$

where A_t is given in Theorem 1.

Proof.- Using the previous notation and the results of Theorem 1 and Lemma 2, we have:

$$a_1 = E_0(\theta_1 \mid D_0, M_A, \delta) = E_0(\theta_1 \mid D_0, M_0) = a_1^0 = a_1^0 + b_1\delta$$

$$f_1 = E_0(Y_1 \mid D_0, M_A, \delta) = E_0(Y_1 \mid D_0, M_0) = f_1^0 = f_1^0 + g_1^{'}\delta$$

$$m_1 = E_0(\theta_1 \mid D_1, M_A, \delta) = a_1 + A_1 (Y_1 - f_1) = a_1^0 + A_1(Y_1^0 + r_1^{'}\delta - f_1^0 - g_1^{'}\delta) = m_1^0 + c_1\delta$$

and so the result is true for t = 1. Then, if we suppose that at time t-1 the result is true, it follows that:

$$a_t = E_0(\theta_t \mid D_{t-1}, M_A, \delta) = G_t m_{t-1} = G_t(m_{t-1}^0 + c_{t-1}\delta) = a_t^0 + b_t\delta$$

$$f_t = E_0(Y_t \mid D_{t-1}, M_A, \delta) = F_t^{'} a_t = F_t^{'} (a_t^0 + b_t\delta) = f_t^0 + g_t^{'}\delta$$

$$m_t = E_0(\theta_t \mid D_t, M_A, \delta) = a_t + A_t(Y_t - f_t) = a_t^0 + b_t\delta + A_t(Y_t^0 + r_t^{'}\delta - f_t^0 - g_t^{'}\delta) = m_t^0 + c_t\delta$$

This establishes (A.9). ■

Lemma 4.- If the model M_A given by (6)-(10) is true, it is verified that:

$$a_t^A = a_t^0 + d_t\delta \qquad f_t^A = f_t^0 + r_t^{'}\delta \quad \text{and} \quad m_t^A = m_t^0 + d_t\delta \tag{A.11}$$

with d_t and r_t defined in Lemmas 1 and 2, respectively.

Proof.- The proof is also made by induction:

$$a_1^A = E_A(\theta_1 \mid D_0, M_A, \delta) = E_A(G_1\theta_0 + H_1\delta + w_1 \mid D_0, M_A, \delta) = G_1 m_1^0 + H_1\delta = a_1^0 + d_1\delta$$

$$f_1^A = E_A(Y_1 \mid D_0, M_A, \delta) = E_A(F_1^{'} \theta_1 + X_1^{'}\delta + v_1 \mid D_0, M_A, \delta) = f_1^0 + r_1^{'}\delta$$

$$m_1^A = E_1(\theta_A \mid D_1, M_A, \delta) = a_1^A + A_1 (Y_1 - f_1^A) = m_1^0 + d_1\delta$$

so the result is true at time 1. If we assume that the result is true for period t-1, then for period t it is verified that:

$$a_t^A = E_A(\theta_t \mid D_{t-1}, M_A, \delta) = G_t m_{t-1}^A + H_t\delta = G_t\left(m_{t-1}^0 + d_{t-1}\delta\right) + H_t\delta = a_t^0 + d_t\delta$$

$$f_t^A = E_A(Y_t \mid D_{t-1}, M_A, \delta) = F_t' a_t^A + X_t' \delta = F_t'(a_t^0 + d_t\delta) + X_t'\delta = f_t^0 + r_t'\delta$$

$$m_t^A = E_A(\theta_t \mid D_t, M_A, \delta) = a_t^A + A_t(Y_t - f_t^A) = m_t^0 + d_t\delta$$

This establishes (A.11). ■

Proof of Theorem 2

Using a similar proof to Theorem 1 it is easily shown that $Y_t \mid D_{t-1}, M_A, \delta, V$ is normal with variance VQ_t^* given that, conditional on δ and V, M_A is a linear state space model similar to the model given by (1)-(4). With regard to the mean, and using Lemmas 3 and 4 it is verified that:

$$E_A(u_t \mid D_{t-1}, V, M_A, \delta) = E_A(Y_t \mid D_{t-1}, M_A, \delta) - f_t = f_t^A - f_t = (r_t' - g_t')\delta = x_t'\delta$$

in such a way that Theorem 2 is established. ■

Proof of Theorem 3.-

a) It can be proved that:

$$P(u_t / D_{t-1}, V, M_0) = \left(2\pi VQ_t^*\right)^{-1/2} \exp\left\{-\frac{u_t^2}{2VQ_t^*}\right\} \qquad \text{(A.12)}$$

$$P(V / D_{r-1}, M_0) = \frac{\left(\frac{n_{r-1}S_{r-1}}{2}\right)^{\frac{n_{r-1}}{2}}}{\Gamma\left(\frac{n_{r-1}}{2}\right)} \exp\left\{-\frac{1}{V}\left(\frac{n_{r-1}S_{r-1}}{2}\right)\right\} V^{-\left(\frac{n_{r-1}}{2}+1\right)} \qquad \text{(A.13)}$$

From (A.12) and (A.13) it follows that:

$$P(u_r, u_{r+1} \ldots, u_t \mid D_{r-1}, M_0) = \int P(u_r, u_{r+1}, \ldots, u_t \mid D_{r-1}, V, M_0) P(V \mid D_{r-1}, M_0) dV =$$

$$= \int \prod_{i=r}^{t} P(u_i \mid D_{i-1}, V, M_0) P(V \mid D_{r-1}, M_0) dV =$$

$$= \left(\prod_{i=r}^{t} 2\pi Q_i^*\right)^{-1/2} \frac{\left(\frac{n_{r-1}S_{r-1}}{2}\right)^{\frac{n_{r-1}}{2}}}{\Gamma\left(\frac{n_{r-1}}{2}\right)} \int \exp\left\{-\frac{1}{V}\left\{\frac{n_{r-1}S_{r-1} + \sum_{i=r}^{t} \frac{u_i^2}{Q_i^*}}{2}\right\}\right\} V^{-\left(\frac{n_{r-1}+(t-r+1)}{2}+1\right)} dV =$$

$$=\left(\prod_{i=r}^{t} 2\pi Q_i^*\right)^{-1/2} \frac{\left(\frac{n_{r-1}S_{r-1}}{2}\right)^{\frac{n_{r-1}}{2}}}{\Gamma\left(\frac{n_{r-1}}{2}\right)} \int \exp\left\{-\frac{1}{V}\left\{\frac{n_t S_t}{2}\right\}\right\} V^{-\left(\frac{n_t}{2}+1\right)} dV \qquad \text{(A.14)}$$

From (A.14), (11) is deduced

b) From Theorem 2 it is verified that:

$$P(u_r,\ldots,u_t / D_{r-1}, V, M_A) = \left(\prod_{i=r}^{t} 2\pi V Q_i^*\right)^{-1/2} \left(\frac{\left|q_t^*\right|}{\left|q_{r-1}^*\right|}\right)^{1/2} \exp\left\{-\frac{1}{2V}\left\{\lambda_{r-1}^{*'} q_{r-1}^{*-1} \lambda_{r-1}^* - \sum_{i=r}^{t} \frac{u_i^2}{Q_i^*} - \lambda_t^{*'} q_t^{*-1} \lambda_t^*\right\}\right\}$$

$$P(V / D_{r-1}, M_A) = \frac{\left(\frac{n_{r-1}S_{r-1}^*}{2}\right)^{\frac{n_{r-1}}{2}}}{\Gamma\left(\frac{n_{r-1}}{2}\right)} \exp\left\{-\frac{1}{V}\left(\frac{n_{r-1}S_{r-1}^*}{2}\right)\right\} V^{-\left(\frac{n_{r-1}}{2}+1\right)}$$

with λ_t^*, q_t^* and S_t^* defined in (12)- (14). The rest of the proof is similar to a) and it is omitted ■

TABLES

TABLE 1.
Comparison of the monitoring and intervention procedures for the Lake Huron series

Procedure	RMSE	MAD	LLF
Without Monitoring	1.1834	0.9591	-1.6358
GS monitor	1.0611	0.8806	-1.4878
WH monitor	1.0904	0.8996	-1.5355

TABLE 2.
Posterior estimations of the shocks vector for the Lake Huron series

Intervention	**$E[\delta \mid D_T, M_0]$**	**$D[\delta \mid D_T, M_0]$**
Outlier 1929	1.2606	0.6099
Level Change 1942	0.6092	0.2257
Slope Change 1942	0.0201	0.0086
Level Change 1943	0.5611	0.2254
Slope Change 1943	0.0181	0.0084
Outlier 1964	-1.3600	0.6115

TABLE 3.
Comparison of the monitoring and intervention procedures for the CP6 series

Procedure	**RMSE**	**MAD**	**LLF**
Without Monitoring	45.1718	36.0134	-5.3811
GS monitor	37.9861	29.1810	-5.1338
WH monitor	40.9156	30.2199	-5.2820

TABLE 4.
Posterior estimations of the shocks vector for the CP6 series

Intervention	**$E[\delta \vert D_T, M_0]$**	**$D[\delta \vert D_T, M_0]$**
Outlier December 1955	74.22	24.04
Level Change January 1956	30.29	14.28
Level Change January 1957	29.05	13.76
Level Change August 1957	-23.13	13.09
Slope Change August 1957	-3.41	1.74

TABLE 5.
Comparison of the monitoring and intervention procedures
for the logarithms of the monthly number of car drivers killed or
seriously injured in the United Kingdom

Procedure	**RMSE**	**MAD**	**LLF**
Without Monitoring	0.1057	0.0797	0.8097
GS monitor	0.0917	0.0708	0.9673
WH monitor	0.1027	0.0789	0.8107

TABLE 6.
Posterior estimations of the shocks vector for the logarithms of the monthly number of car drivers killed or seriously injured in the United Kingdom

Intervention	$E[\delta\|D_T,M_0]$	$D[\delta\|D_T,M_0]$
Level Change October 1973	-0.0510	0.0193
Slope Change October 1973	-0.0014	0.0006
Level Change November 1973	-0.0516	0.0192
Slope Change November 1975	-0.0014	0.0006
Level Change January 1975	-0.0405	0.0180
Slope Change January 1975	-0.0008	0.0005
Outlier August 1976	-0.1262	0.0616
Level Change January 1983	-0.0392	0.0148
Level Change February 1983	-0.0415	0.0142
Slope Change February 1983	-0.0006	0.0003

TABLE 7.
Comparison of the monitoring and intervention procedures
for the food industry sales series

Procedure	RMSE	MAD	LLF
Without Monitoring	0.7524	0.5473	-1.2043
GS monitor	0.7252	0.5400	-1.1510
WH monitor	0.6902	0.5125	-1.2240

TABLE 8.
Posterior estimations of the shocks vector
for the food industry sales series

Intervention	$E[\delta \vert D_T, M_0]$	$D[\delta \vert D_T, M_0]$
Level Change April 1978	0.4110	0.1686

FIGURES

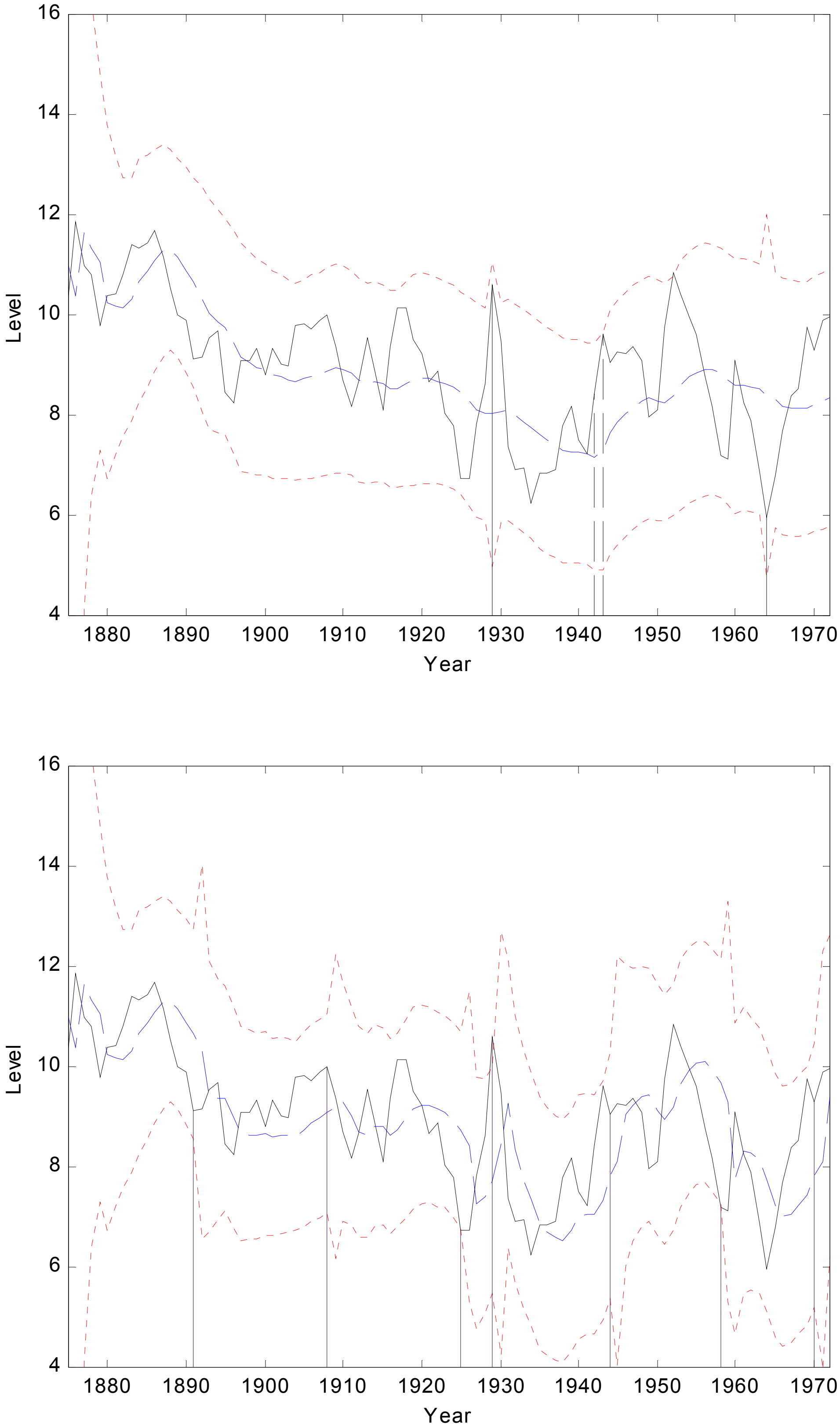


Figure 1: One-step ahead forecast (dashed line) and 99% uncertainty limits (dotted lines) for the lake Huron series (continuous line). **Above**: with GS monitor, **below**: with WH monitor

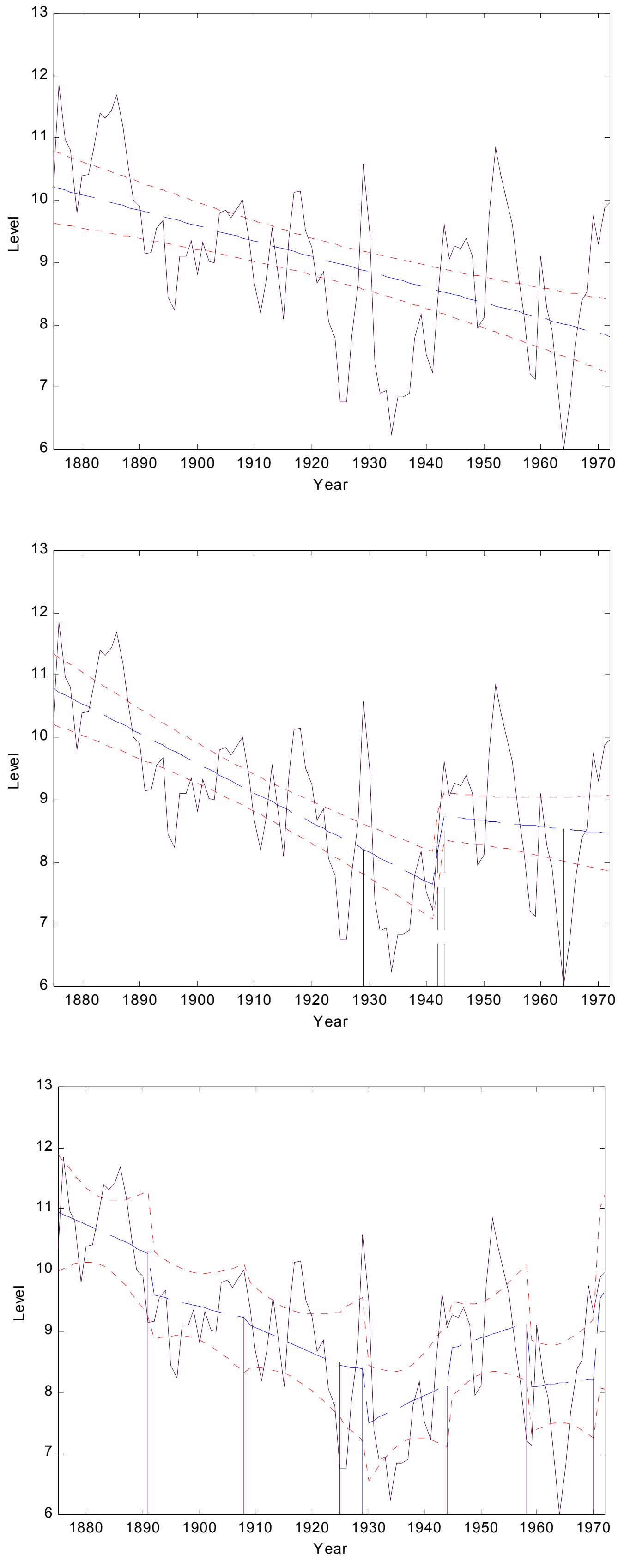


Figure 2: Retrospective estimations of the trend (dashed line) and the 99% uncertainty limits (dotted lines) for the lake Huron series by applying the three procedures. **Above**: without monitoring (dotted lines), **middle:** with WH monitor and **below** with GS monitor

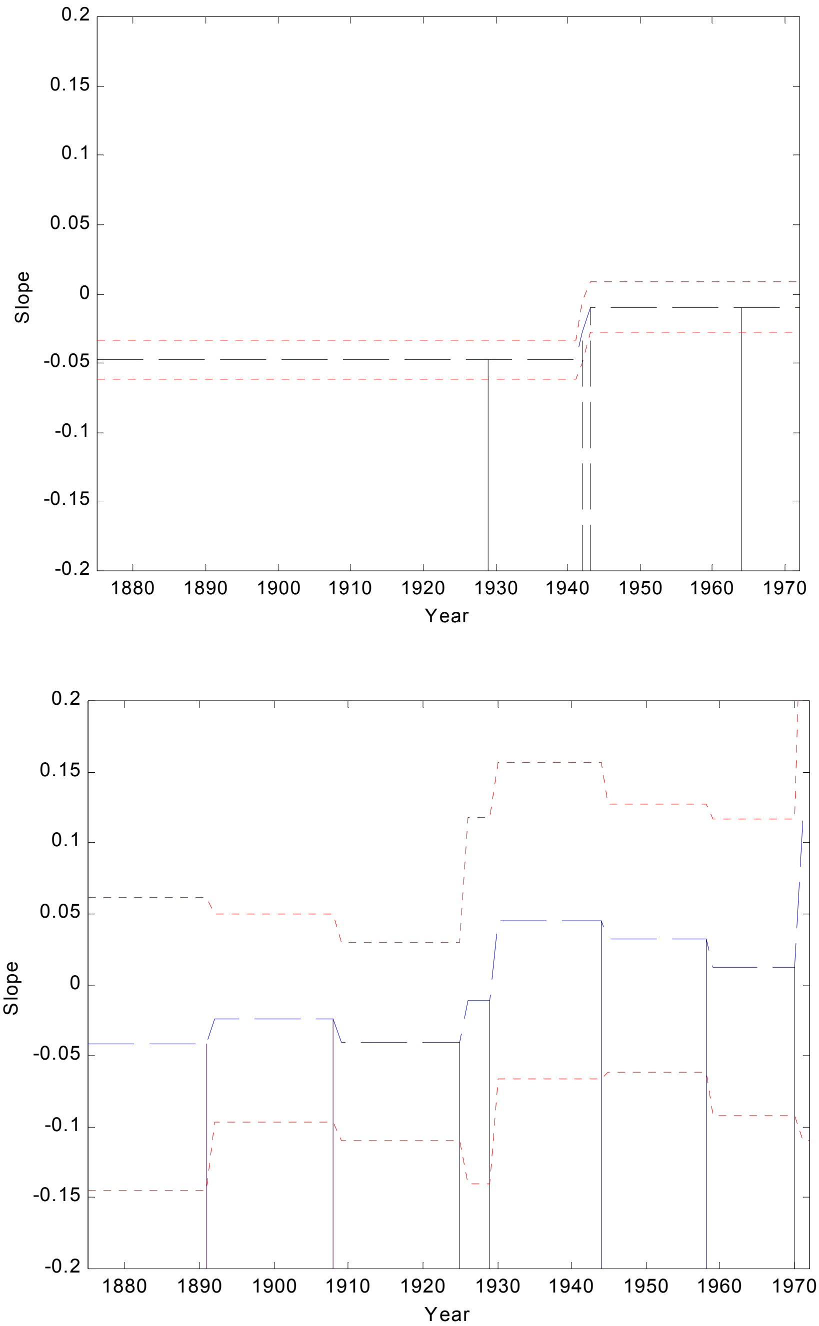


Figure 3: Retrospective estimations of the slope (dashed line) and the 99% uncertainty limits (dotted lines) for the lake Huron series by applying the three procedures. **Above:** with GS monitor and **below** with WH monitor

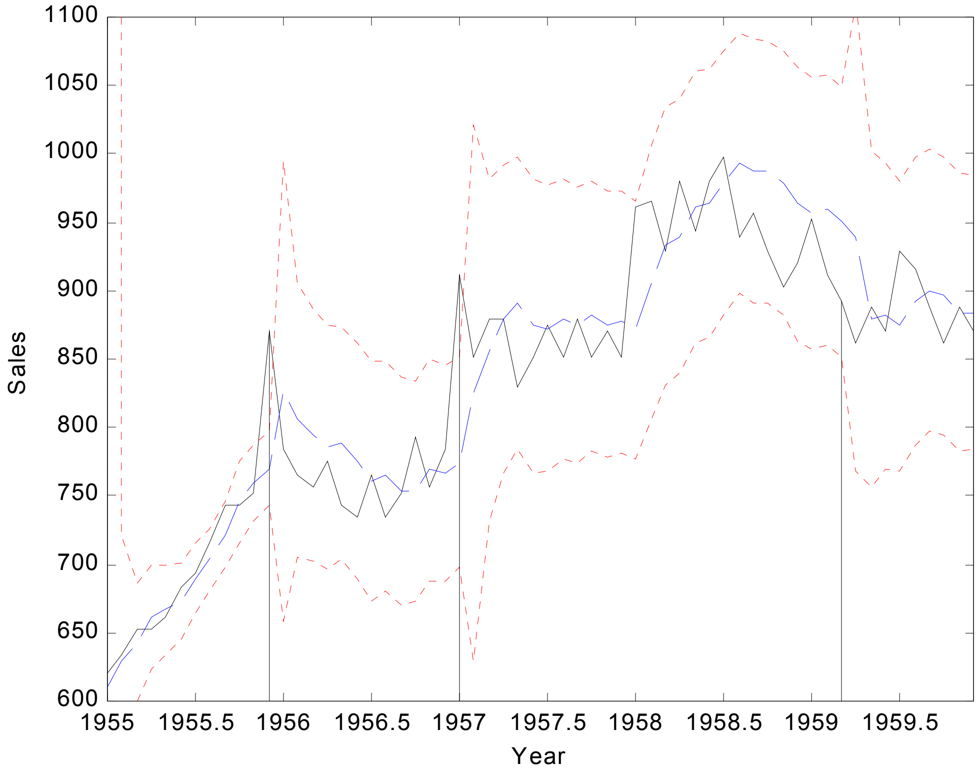


Figure 4: One-step ahead forecast (dashed line) and 99% uncertainty limits (dotted lines) for the CP6 series (continuous line). **Above**: with GS monitor, **below**: with WH monitor

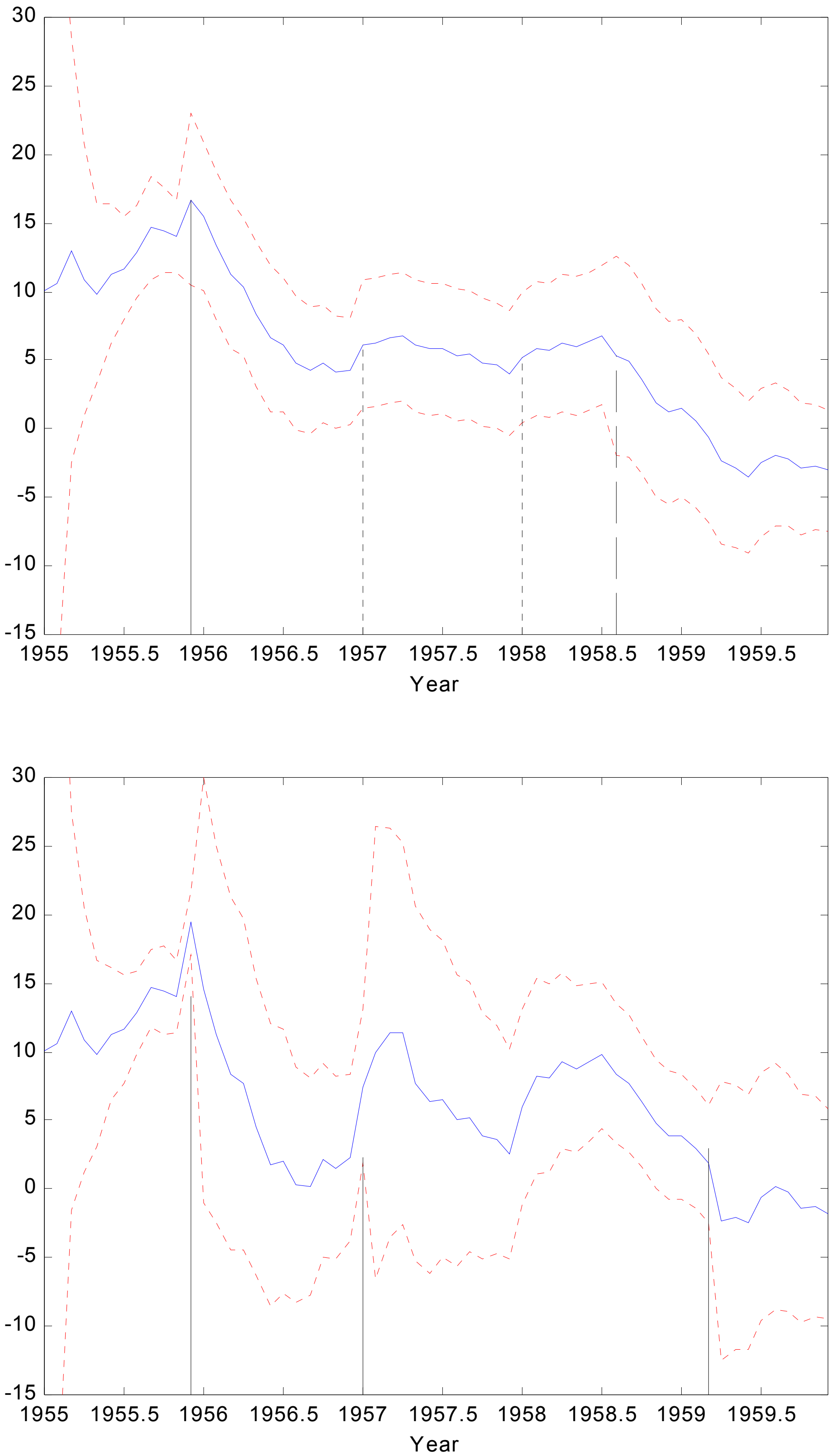


Figure 5: On line estimation of the slope (dashed line) and 99% uncertainty limits (dotted lines) for the CP6 series (continuous line). **Above**: with GS monitor, **below**: with WH monitor

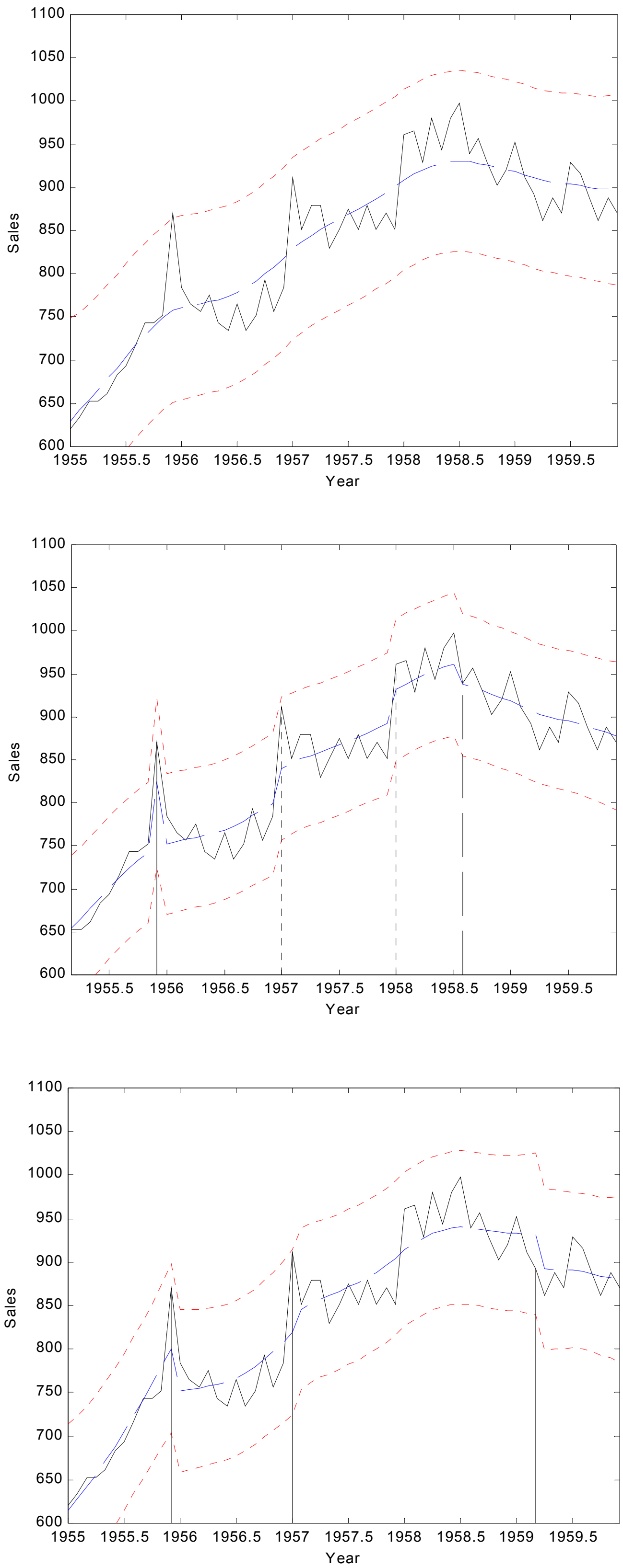


Figure 6: Retrospective estimations of the trend (dashed line) and the 99% uncertainty limits (dotted lines) for the CP6 series by applying the three procedures. **Above**: without monitoring (dotted lines), **middle:** with WH monitor and **below** with GS monitor

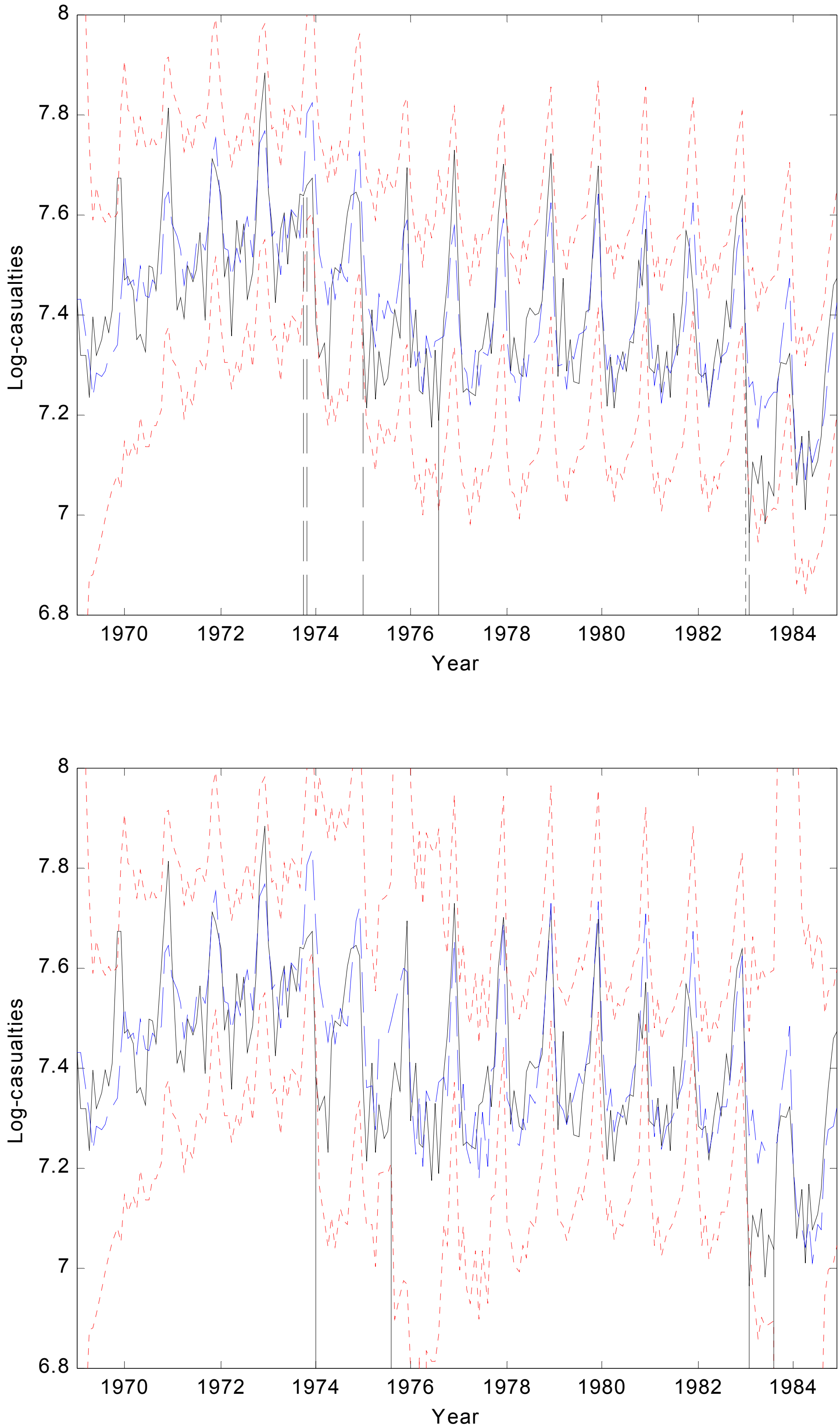
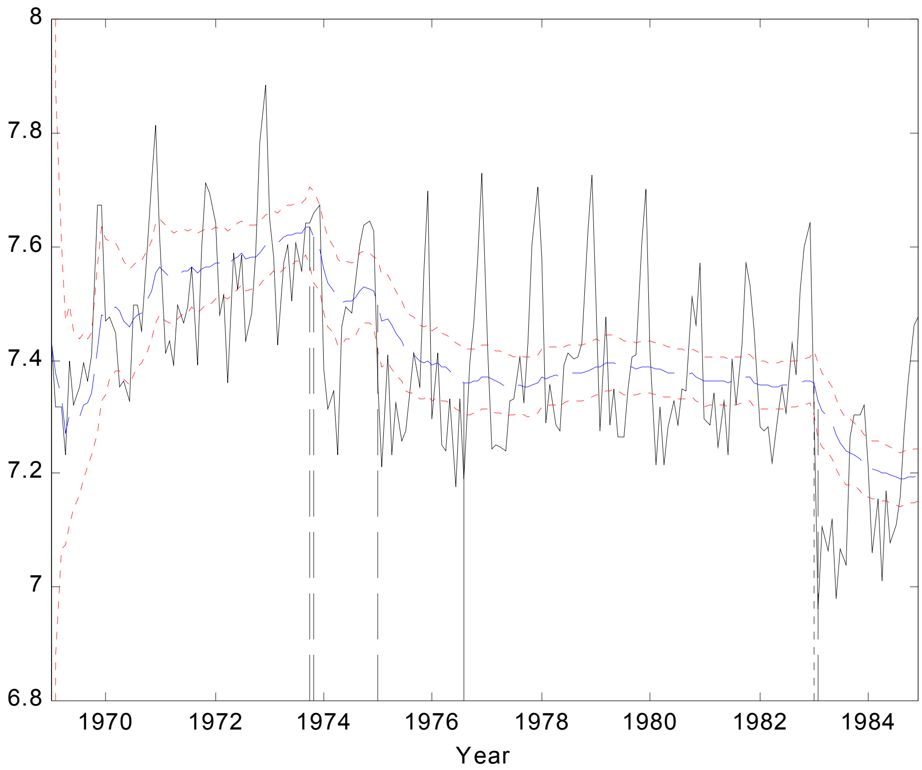


Figure 7: One-step ahead forecast (dashed line) and 99% uncertainty limits (dotted lines) for the logarithm of the monthly number of car drivers killed or seriously injured in the United Kingdom (continuous line) obtained by applying the three procedures. **Above**: with GS monitor, **below**: with WH monitor



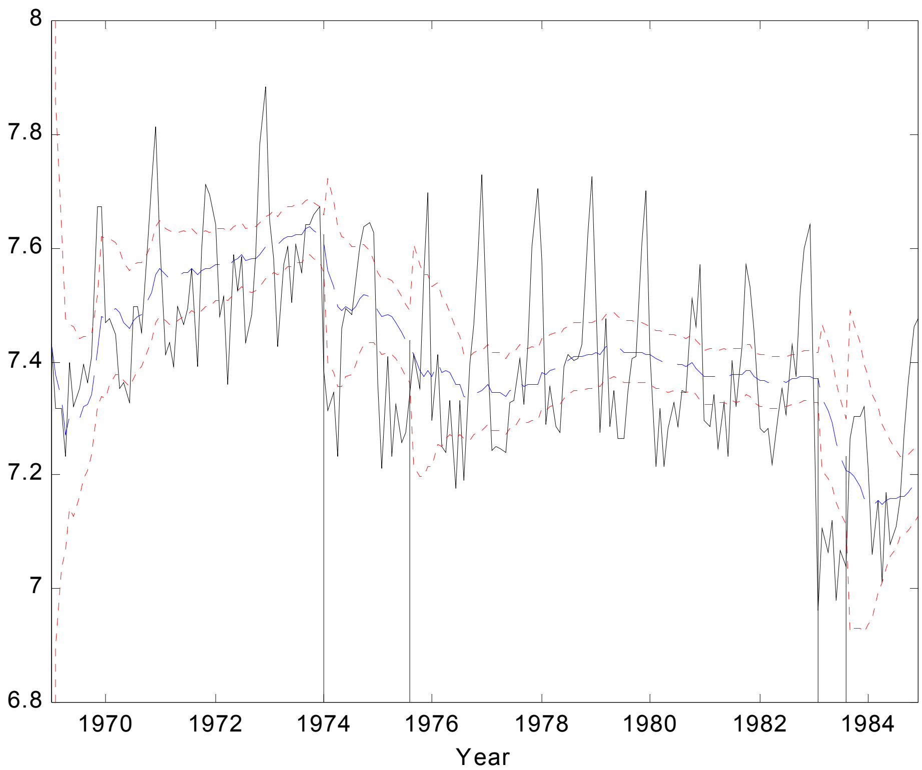
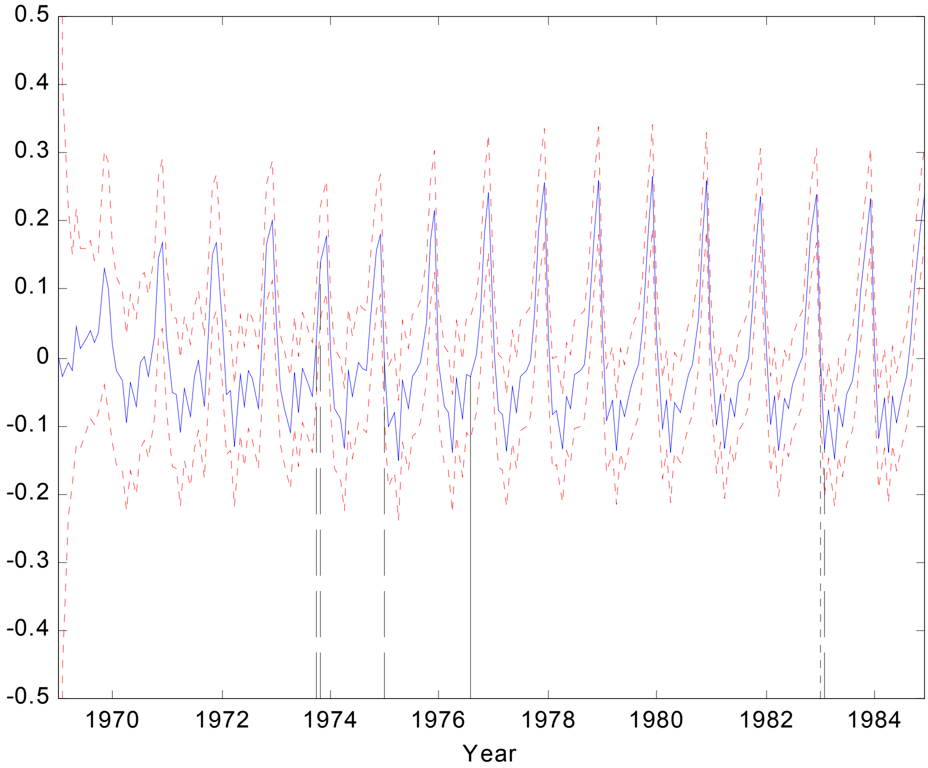


Figure 8: On line estimation of the trend (dashed line) and 99% uncertainty limits (dotted lines) for the logarithm of the monthly number of car drivers killed or seriously injured in the United Kingdom (continuous line) obtained by applying the three procedures. **Above**: with GS monitor, **below**: with WH monitor



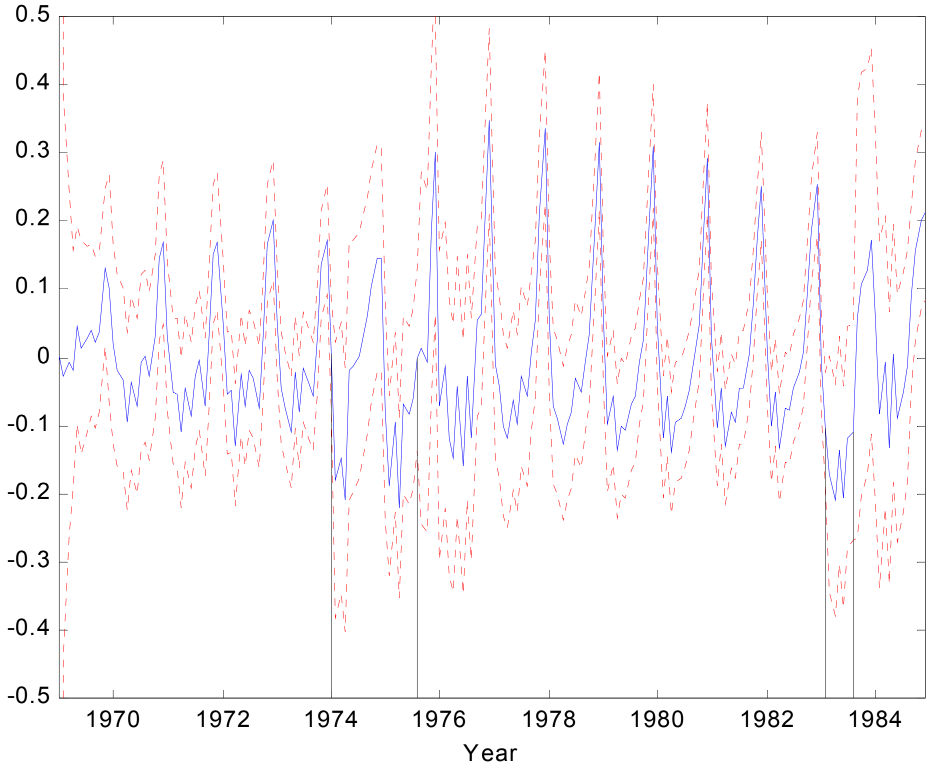


Figure 9: On line estimation of the seasonal variation (dashed line) and 99% uncertainty limits (dotted lines) for the logarithm of the monthly number of car drivers killed or seriously injured in the United Kingdom (continuous line) obtained by applying the three procedures. **Above**: with GS monitor, **below**: with WH monitor

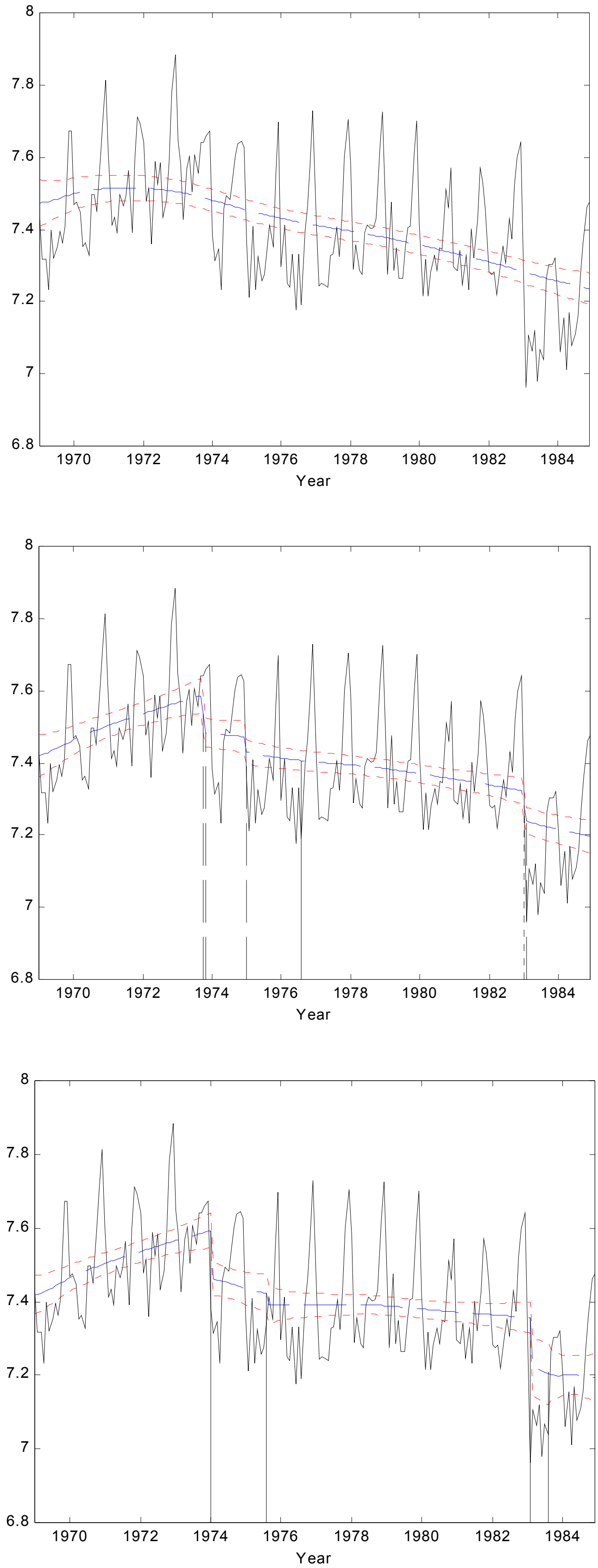
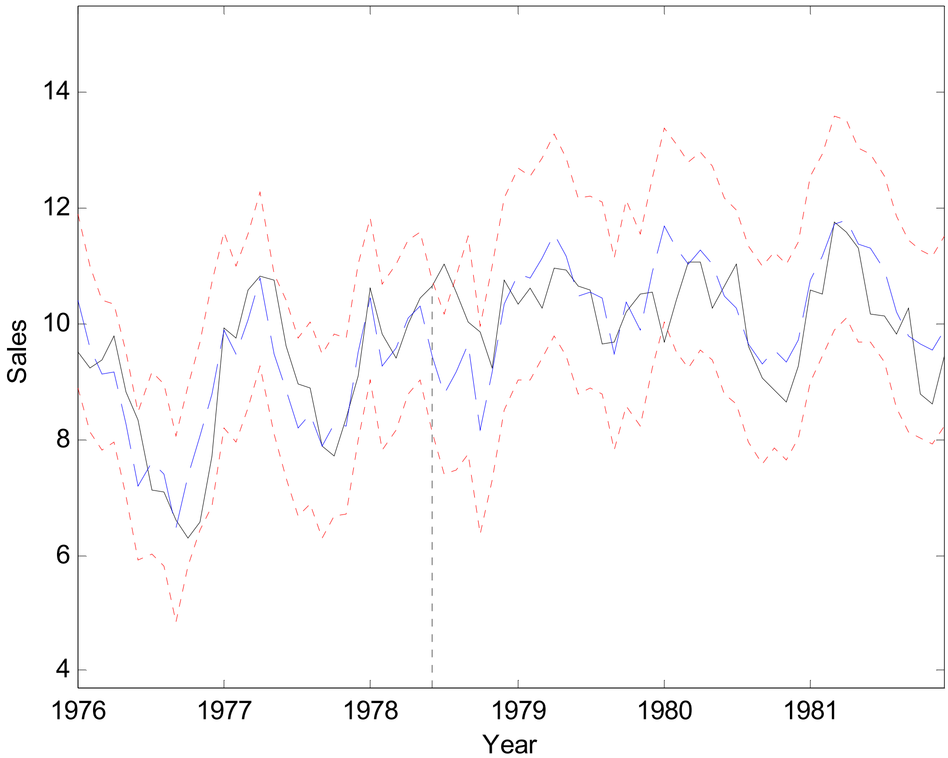


Figure 10: Retrospective estimations of the trend (dashed line) and the 99% uncertainty limits (dotted lines) for the logarithm of the monthly number of car drivers killed or seriously injured in the United obtained by applying the three procedures. **Above**: without monitoring (dotted lines), **middle:** with WH monitor and **below** with GS monitor



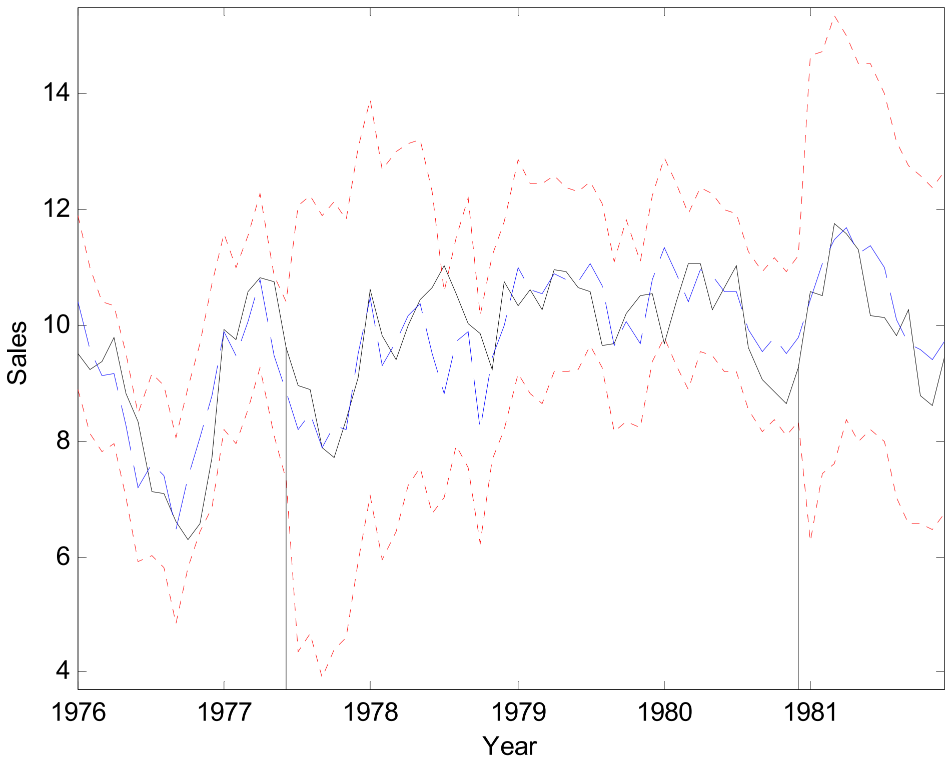
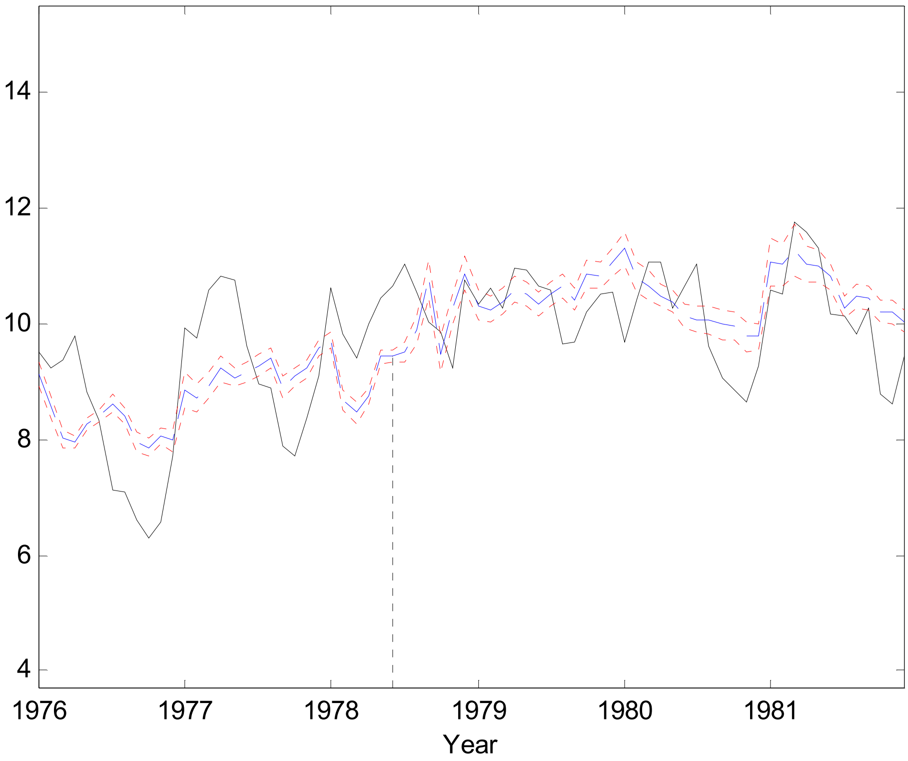


Figure 11: One-step ahead forecast (dashed line) and 99% uncertainty limits (dotted lines) for the food industry sales series (continuous line) obtained by applying the three procedures. **Above**: with GS monitor, **below**: with WH monitor



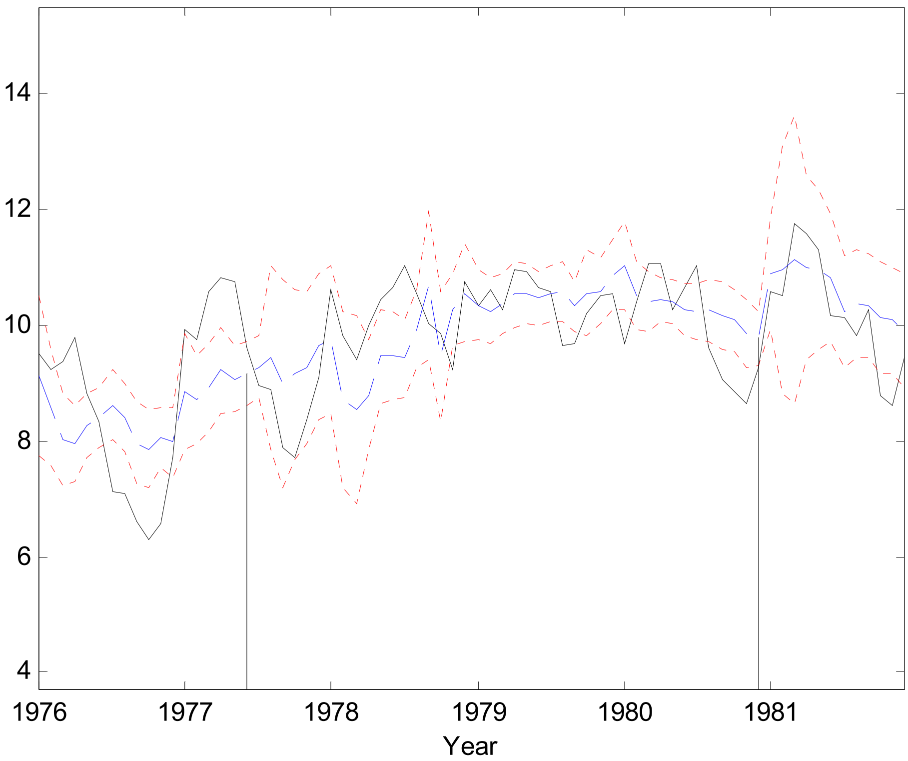


Figure 12: On-line non-seasonal trend estimation (dashed line) and 99% uncertainty limits (dotted lines) for the food industry sales series (continuous line) obtained by applying the three procedures. **Above**: with GS monitor, **below**: with WH monitor

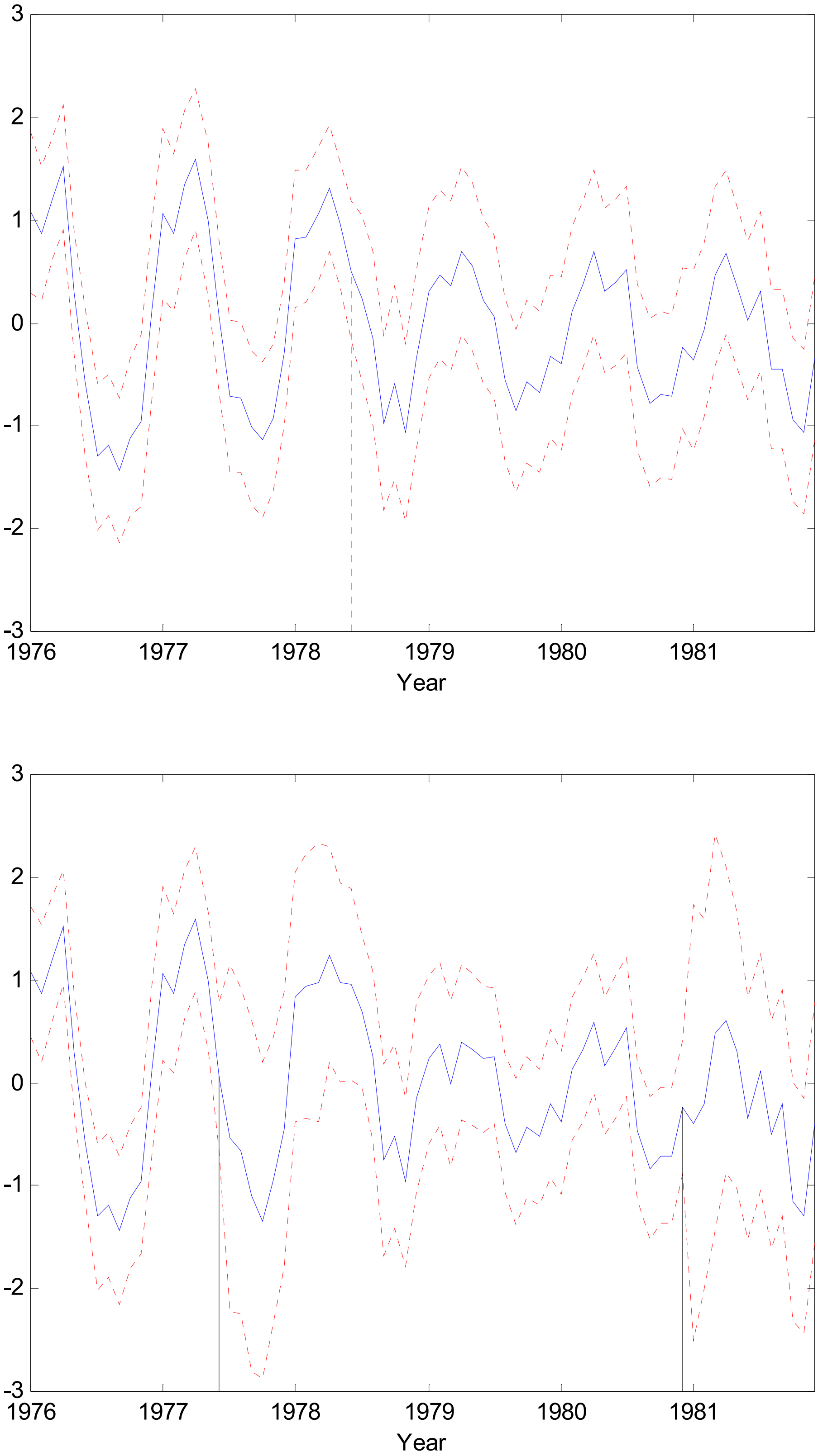


Figure 13: On-line non-seasonal pattern estimation (dashed line) and 99% uncertainty limits (dotted lines) for the food industry sales series obtained by applying the three procedures. **Above**: with GS monitor, **below**: with WH monitor

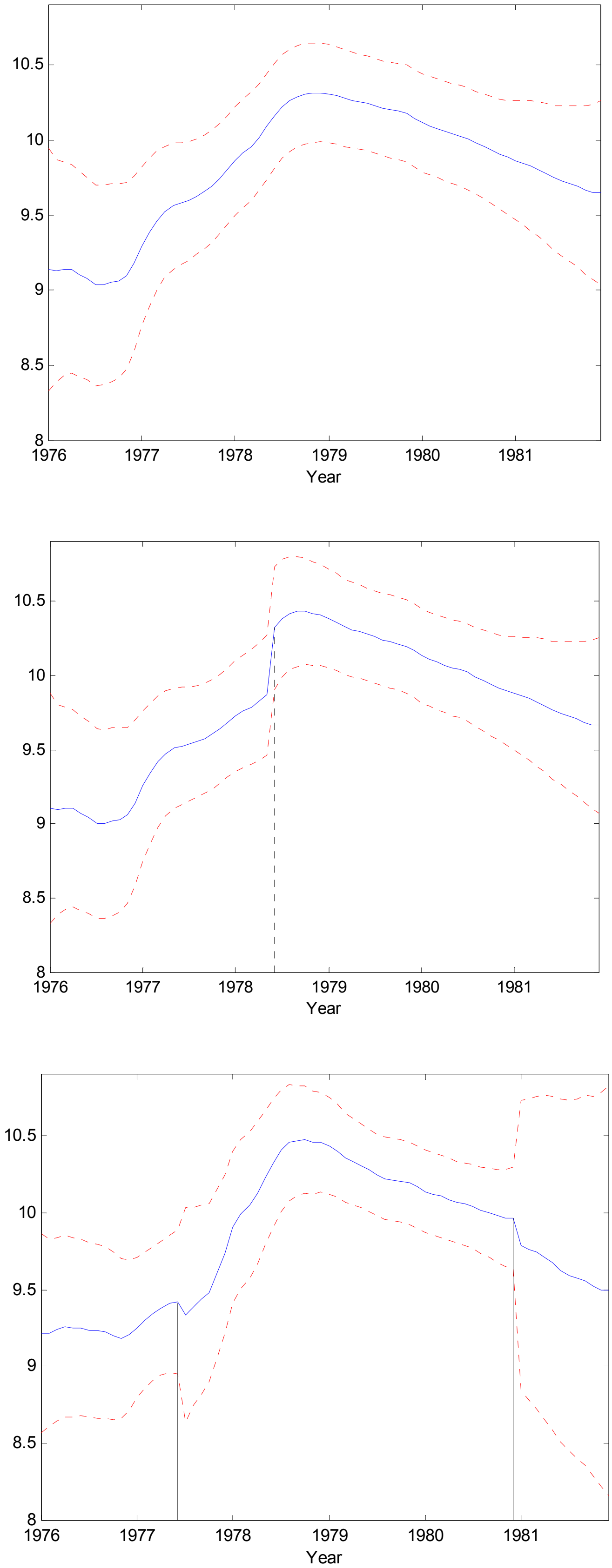


Figure 14: Retrospective estimation of the intercept (continuous line) and 99% uncertainty limits (dotted lines) for the food industry sales series obtained by applying the three procedures. **Above**: without monitoring, **middle:** with GS monitor, **below**: with WH monitor

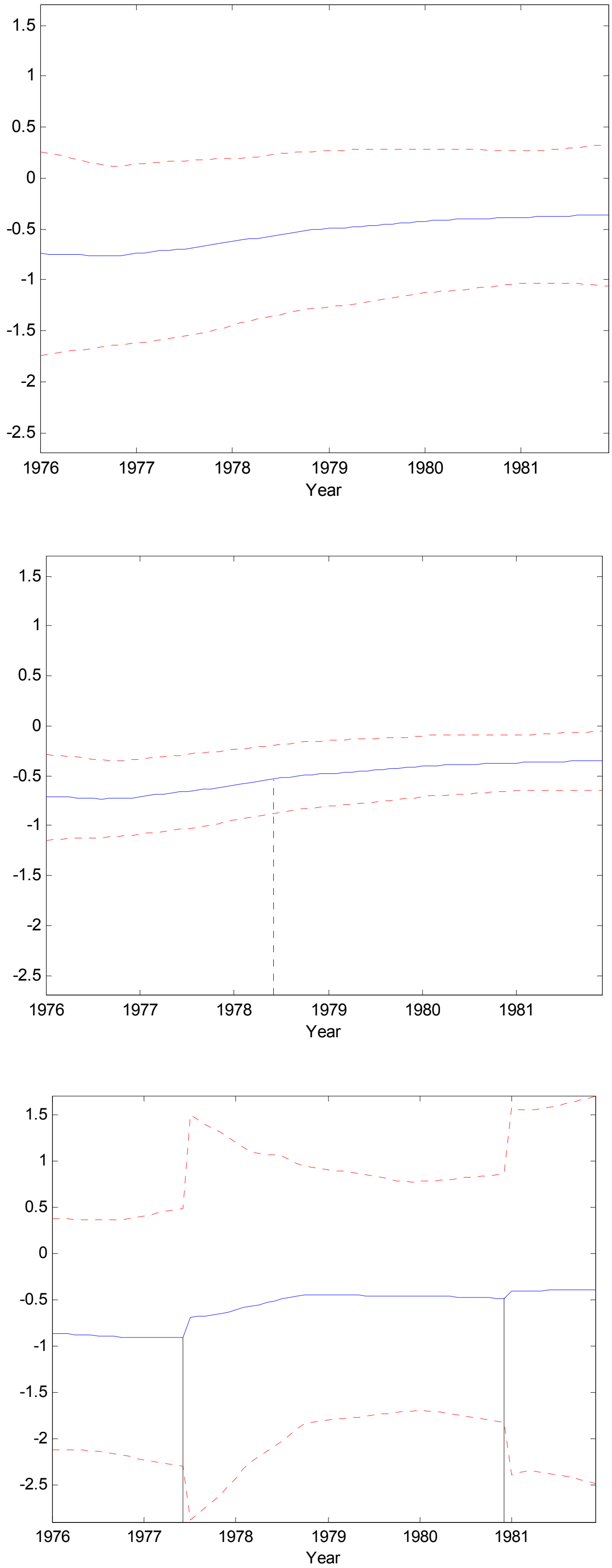


Figure 15: Retrospective estimation of the regression coefficient (continuous line) and 99% uncertainty limits (dotted lines) for the food industry sales series obtained by applying the three procedures. **Above**: without monitoring, **middle:** with GS monitor, **below**: with WH monitor

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